ABOUT THIS BOOK

This book is a simple and largely non-technical account of aerodynamics, a branch of science which has developed greatly in recent years and which lies at the root of all problems of mechanical flight. It traces the story of the conquest of the air, and the slow unfolding of the secret of flight, from the spears and arrows of the primitive hunter to the supersonic aircraft and the giant rocket of to-day. It tells how the classical hydrodynamics reached an *impasse* which was ultimately resolved by patient work in the wind tunnel and the brilliant inspiration of the boundary-layer theory; how Lanchester discovered the real secret of the aircraft wing long before the Wrights flew, and was not believed, and finally, how the ancient Chinese invention of the rocket has emerged again, perhaps this time to conquer space as the aeroplane has conquered the air.

Throughout the book, the aim of the author has been to explain the ideas which lie behind the mathematical theory and the difficulties which beset its path in terms which can be understood by the layman, and so bring the reader, by easy stages, within sight of the frontiers of research.

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THE SCIENCE OF FLIGHT

O. G. SUTTON

The Science of Flight

BY

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To my wife

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Preface

As its title indicates, this is a book about the theoretical aspects of flying. It is not concerned with the problem of aircraft design, which is the province of the engineer, and nowhere does it touch upon the actual technique of flying. In short, it is an attempt to explain to the layman something of a branch of applied mathematics, called aerodynamics, which lies at the root of all matters appertaining to mechanical flight. I have written the book because I believe there are many people who would like to know more of these matters.

This means putting into plain English much which is usually expressed in the language and notation of mathematics, an interesting but never very easy task for the author who must, of necessity, be steeped in that notation himself. The difficulty here, I think, is precisely that which faces a judge when he has to sum up in an intricate case and explain the law to the jury. It is not that the legal argument is too involved for the jury to grasp but rather that law, like mathematics, takes the words of everyday speech and uses them for a special purpose and with limited and, frequently, archaic meaning. Mathematical language has the same legacy from the past and at times seems to take a perverse delight in distorting the meanings of commonplace words – thus 'imaginary' numbers are as real as the figures on a cheque, a 'vortex' does not necessarily mean a whirlpool and a 'shock wave' is not a wave in the familiar sense of the word, nor is there anything shocking about it.

It seems to me that mathematics in general, and applied mathematics in particular, must be capable of being explained without recourse to its own peculiar symbols and language. This is not to say that the traditional notation can be dispensed

with, for without it progress in mathematics would hardly be possible, but in any physical science equations and formulæ must surely be less important than the ideas which they embody. From the reader of this book I ask nothing more than an acquaintance, however neglected, with the elements of algebraic notation, and then only, when to express a simple mathematical formula in words would be pedantic and confusing. In the book there are plenty of difficult and probably unfamiliar ideas, many of which took mankind centuries to reach, but none, I trust, which are not clear of themselves.

I hope I have nowhere given the idea that the problems of fluid mechanics and of aerodynamics in particular can be explained away in a few easy phrases, because I believe this to be impossible, and anyway, the subject would hardly be worth getting excited about if it were as simple as that. The principle I have endeavoured to follow in reducing these matters to ordinary language is that of writing the truth and nothing but the truth, but not always the whole truth, even where I can persuade myself that I know it. To do so would be to produce a precis of a text-book, and I have no wish to pretend that it is possible to condense the whole of fluid mechanics into so short a compass. My aim throughout has been to show the broad pattern of the science rather than the detail, not as something finished and finite but as it is even to-day unrolling from the loom.

I am greatly indebted to my colleagues, Professor G. D. West and Mr S. D. C. Munday, who have kindly read this book in manuscript and whose suggestions have been most helpful. To Messrs Constable and Company I owe my thanks for their permission to reproduce certain pages from Lanchester's original books, and I am indebted to the Directors of the National Physical Laboratory and of the Science Museum for permission to reproduce certain photographs. Finally, I would like to express my gratitude to my wife for her assist-

ance in reading and checking the manuscript and to my secretary, Mrs F. D. Crockford, for her care and patience in preparing the typescript.

Shrivenham,
July 1948

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CHAPTER I

The Beginnings of Aerodynamics

AERODYNAMICS is the science which records, analyses and makes amenable to calculation the complex phenomena on which flight depends. The emergence of aerodynamics as a logical and exact study is of comparatively recent origin, and while the foundations of most sciences had been securely laid by the end of the eighteenth century, real understanding of the principles upon which practical aircraft can be designed has only been achieved in the last fifty years or so.

The histories of the exact sciences usually show a familiar and easily recognized pattern. Initially there is a period of unguided and unrelated experimentation when the simpler and more striking facts are discovered, often in a dramatic or unexpected fashion. At this stage the experimenters are working in the dark and any theories which may be formulated are necessarily speculative and, as often as not, utterly wrong. This period is generally followed by an era of consolidation, particularly on the experimental side, with results becoming more precise and numerical, if less spectacular, than before and a definite technique of experimentation begins to emerge. This stage is almost always characterized by an abundance of empirical formulæ which have been invented, in default of a general theory, as a convenient way of summarizing results and as guides for future work. The science reaches maturity when the experimental probings have gone deep enough to reveal the true theoretical basis of the subject; the empirical formulæ give place to exact mathematical theorems, which may ultimately attain the status of 'natural laws' whose truth is accepted without question by all concerned.

Fluid mechanics, of which aerodynamics is a part, has not entirely conformed to this familiar pattern of development. In mechanics and astronomy, and later in electricity and magnetism, optics, heat and thermodynamics, and other branches of mathematical physics, the calculus of Newton and Leibnitz opened the way to swift and sure progress with theory and observation only rarely in conflict. In the eighteenth century Euler and Bernoulli applied the calculus to problems of fluid motion and so founded the classical hydrodynamics, which deals with motion in a hypothetical medium called an 'ideal fluid'. The classical hydrodynamics became a subject of immense attraction to mathematicians who made it so highly abstract as almost to deserve the name of 'pure' mathematics. To engineers it had considerably less appeal, for they found its results either unintelligible or completely at variance with actuality when applied to real fluids. In the eighteenth and nineteenth centuries the classical hydrodynamics, following its aim of producing a logical and consistent theory capable of yielding exact solutions of idealized problems, became a largely academic study and made little effective contribution to the problem of flight.

For the solution of practical problems, such as the determination of the pressure-drop in pipes or the resistance experienced by a body moving through a fluid (neither of which can be satisfactorily solved by the classical hydrodynamics), engineers invented the pseudo-science of hydraulics, the aim of which was to solve real problems approximately. Without an underlying unified theory hydraulics soon became a morass of empirical or semi-empirical formulæ designed to solve individual and usually unrelated problems. To the mathematician it was merely a 'science of coefficients'.

Aerodynamics as we now know it and as it will be described in this book represents a kind of fusion of these two dissimilar approaches. In many respects, but not in all, the atmosphere

behaves like the hypothetical ideal fluid of the classical hydrodynamics, but no real aircraft could be designed on this basis alone. It is necessary to bring in some, at least, of the empiricism of hydraulics, often with the happiest results. To-day the position is that the aeronautical engineer is able to profit from the discipline of hydrodynamics without losing sight of reality, and to use the practical results of hydraulics with the certainty that his formulæ have a sound theoretical basis.

To comprehend a science is to know something of its history as well as to be versed in its technique. Aerodynamics has a particularly fascinating story, which might almost be described as a puzzle in which the essential clues were not found until a very late stage. The story of the solution is told in the pages which follow, but to appreciate what it has all meant we must first go back to the time when primitive man hunted his dinner, and only birds and insects flew.

THE EARLIEST PROJECTILES

We cannot know with any certainty when man first deliberately shaped his weapons for throwing, but that act of conscious design marked the first step on a road which leads from the spear and the arrow to the aeroplane and the giant rocket of to-day. The urge to throw things seems, in fact, to be one of the most elementary and deep-seated of our instincts, appearing in childhood and persisting into old age. The more mature ambition to throw or propel things swiftly and accurately, which is the mainspring of most outdoor games, probably has its roots in the ages when the possession of a suitable weapon and the ability to hurl it with force and precision meant the difference between eating or starving, or even life and death if the target was a large and active beast or an equally well equipped enemy.

It is significant that such weapons were conceived and

virtually brought to perfection at a relatively early stage in history – it is doubtful if to-day we could improve on the bows and arrows which won the battles of Crecy and Agincourt, if we were restricted to the same materials. Aerodynamic design in this case is a consequence of the use to which the implement is put and it would be difficult to go seriously wrong in designing a throwing spear *ab initio.* The ultimate object is to inflict as deep a wound as possible, so that the point must be made hard and sharp, usually necessitating the fitting of a flint or metal blade. This combination of light wooden shaft and heavy head means that the main mass of the spear is placed well forward, a fact which has important consequences on its efficiency as a projectile. All well designed projectiles, whether they be a bushman's arrows or a modern shell, must satisfy two requirements; they must land point foremost, that is, must not turn over and over in their flight, and they must always follow the same path when launched in the same way, for without this marksmanship is impossible. In scientific language, a projectile must be stable in its flight, and with a long unrotated projectile, such as a throwing spear, an essential condition for stability is that the centre of gravity shall be kept well forward, as any schoolboy knows when he makes a paper dart. The schoolboy achieves stability – and generally earns retribution – by fixing a pen nib to the forward end of his dart, and the savage who fastens a heavy blade to the end of a long shaft will not be slow in discovering its virtues as a projectile. Nowadays we would say that such a weapon has good ballistics.

Arrow design represents another step forward. The arrow is the earliest true projectile, capable of maintaining direction over distances considerably greater than those achieved with a throwing spear. The addition of feathers to the shaft may have been, in the first instance, a form of sympathetic magic, an attempt to confer swift and accurate flight by giving the

weapon some of the attributes of a bird. Be that as it may, the result is a body of simple but sound aerodynamical design, for the feathers act as control surfaces whose function is to provide a restoring force whenever the projectile starts to deviate from a straight path. The same principle is followed to-day, from the 'flights' of the darts in the village pub to the control fins on the giant V2 rocket.

There is one notable exception to the above instances that primitive weapons more or less dictated their own design. The boomerang must presumably be counted a primitive weapon – it is at any rate the chosen weapon of a very primitive section of the human race – but its design is anything but intuitive. This almost certainly accounts for the fact that its appearance was confined to one portion of the globe, whereas arrows and spears seem to be universal.

ARISTOTLE AND THE MOTION OF PROJECTILES

The ancient Greeks had a number of fanciful tales of human flight, the best known of which is the fable of Dædalus and the wings made for Icarus 'who flew too near the sun'. There is also the account of Archytas of Tarentum and his flying mechanical bird, but the earliest serious analysis of flight occurs in the *Physica* of Aristotle (384–322 B.C.) as a part of his proof of the impossibility of a vacuum. The 'proof' is worthy of consideration because it is a vivid illustration of the gulf which sometimes lies between the ancient Greek mind and our own in the domain of physical science.

Briefly, Aristotle argues that a body such as an arrow can continue in motion only as long as force is continually applied to it and that any withdrawal of the force would immediately cause the body to stop. (This view should be contrasted with that advanced over twenty centuries later by Newton, who based his mechanics on the postulate that a body would con-

tinue in motion unless a force were applied to stop it.) Further, since it was held to be impossible to conceive of anything in the nature of action at a distance, the force required some material medium in contact with the body for its transmission. Thus a projectile could not move in a vacuum and hence a vacuum cannot exist.

The argument clearly requires that the air sustains the projectile in its flight, but Aristotle did not linger over this, to us the vital question. He suggested that the atmosphere might push the arrow along by rushing to fill the vacuum in its rear, or (rather more obscurely) that motion, once started, would be maintained by the air as a consequence of its fluidity. In both cases the air sustained and did not retard the flight of the arrow.

In these arguments lie the characteristic features of the strength and the weakness of ancient Greek science. The strength lies in the power of conceiving a philosophical problem as an abstraction from a material phenomenon, a process which is still an essential feature of all mathematical physics. (In pure mathematics, which is entirely abstract, there is nothing like the gulf which exists between the Greeks and ourselves in applied science.) The whole argument, however, collapses and proves nothing because its premises are at variance with the essential physical facts, and this because of the absence of experimental data from Greek science. Such, however, was the authority of the Aristotelian doctrines that these views held sway, or were given serious consideration, right up to the middle ages of European culture.

LEONARDO DA VINCI'S AERODYNAMICS

The centuries which lie between Aristotle and the birth of modern science with Galileo and Newton hold little of interest for aerodynamics until we come to Leonardo da Vinci (1452–

1519), usually remembered as the painter of what is perhaps the world's best known picture, and less as a gifted engineer who would have found much to occupy him in that great age of mechanical invention, the nineteenth century.

The attitude of mediæval times to flying is amply illustrated by one of its favourite books, the fabulous life of Alexander the Great by the Pseudo-Callisthenes. Alexander is said to have made a flying-machine by harnessing to a yoke two strong eagles which had been kept fasting for three days. When Alexander took his seat on the yoke the eagles flew up with him, following the direction indicated by his spear, at the head of which was a large lump of liver. This tale seems to have struck the fancy of the time and representations in sculpture in St Mark's, Venice, and in the Cathedral of Basle, as well as illustrations in manuscripts (sometimes with eight eagles) are evidences of its popularity. Nor did the same age see anything manifestly absurd in the Tartar and Chinese legend of the Bronze Horse, so graphically described in Chaucer's *Squire's Tale*. Leonardo's massive common sense and shrewd engineering insight come strangely into this world of fable; here at any rate was one man of whom we might say that he at least understood in part not only how birds fly but also how man might fly.

Leonardo discarded the central concept of the Aristotelian scheme, that the air assisted the motion, and instead considered the atmosphere as a resisting medium. This is the essential step, without which all aerodynamic theory would be in vain. Three centuries later, in 1809, Sir George Cayley defined the then unsolved problem of mechanical flight as that of making a surface support a weight by the application of power to the resistance of the air, a definition which is as sound to-day as it was remarkable at the time it was made. Leonardo's concept is exactly this, but he went wrong in his subsequent development of the idea. He supposed that the

flapping motion of the wing of a bird causes the air in contact with it to 'condense' and behave as a rigid body on which the bird is supported, the motion of the wing being sufficiently rapid to ensure that the stroke is completed before the local 'condensation' is passed on to other layers of air. Soaring flight, in which the wings are held nearly motionless, Leonardo explained on the same hypothesis by saying (quite correctly) that what mattered was the relative motion of the air and the wing so that, given a favourable wind, the bird can soar without beating its wings.

A 'condensation' process resembling that envisaged by Leonardo does occur in nature, but only becomes appreciable at very high speeds. What Leonardo did not know was that to attain any noticeable amount of local compression in a free atmosphere it would be necessary for the wings to be moving through the air at extremely high speed, so that his explanation could not possibly apply to, say, the flight of an eagle or any other bird which uses slow powerful wing beats. On the other hand his conception of the bird being able to fly because, and only because, the air offers resistance is essentially correct and marks a great step forward on the Aristotelian doctrine.

With Leonardo also emerges for the first time a reasonable picture of what is entailed in sustaining a man in the air. From an examination of the anatomy of birds he appears to have concluded that a relation exists between wing span and the square of the body weight, and that on this hypothesis a man of normal weight would need wings of some 12 yards span to support him. In making this estimate Leonardo was more realistic than many of those who followed him at a much later date.

Leonardo's ideas, however, played no part in the subsequent development of mechanical flight because they were never made known to his contemporaries in an ordered form. The full magnitude of his achievements was, in fact, not dis-

closed until 1930 when his fellow countryman Giacomelli edited and published the mass of notes he left behind. It is doubtful, however, if publication in the sixteenth century would have had much effect; Leonardo was a man in advance of his time and the gross superstition of his age was not favourable to scientific speculation.

FLUID MECHANICS IN MODERN SCIENCE

Galileo (1564–1642), the forerunner of Newton and the herald of modern science, appears in the story only to give the final death-blow to the Aristotelian theory of the action of air in sustaining motion. Galileo, before Newton, formulated the idea of motion persisting of its own and clearly recognized the essentially dissipative action of air in resisting motion. His attempts to make precise the way in which resistance changes with velocity – a fundamental question in the science of ballistics which was beginning to arise with the invention of firearms – were less successful. Galileo's experimental technique was not adequate for the purpose he had in mind, but his law, that resistance varies directly as the velocity, although not valid for his application, was at any rate a recognition of the important fact that resistance increases with velocity.

Newton (1642–1727) was born in the year that Galileo died. With him enters not only the whole of modern mechanics but also the calculus, perhaps the most powerful and subtle instrument of reason ever devised. From this point onwards the means necessary for the analysis of the problem of flight lay within man's grasp, but it is an ironical fact that the very power and subtlety of the calculus were in many ways responsible for the development of the theoretical aspects of fluid mechanics along lines which tended to diverge from reality. Leonhard Euler (1707–83) and Daniel Bernoulli (1700–82) laid the foundations of the classical hydrodynamics and made

clear the nature and importance of fluid pressure; D'Alembert, Lagrange, Helmholtz, Kelvin, Rayleigh and others brought the study of the dynamics of an ideal fluid to a remarkable degree of mathematical perfection, but without making any direct contribution to the solution of the many practical problems of real fluids, which in fact could not even be discussed in the narrow logic of the classical hydrodynamics.

In the present century fluid mechanics has become considerably more realistic, but at the expense of subdivision into a number of distinct branches. *Highly viscous liquids*, that is, those which exhibit a large resistance to deformation (such as tar or treacle) occupy one branch. With these fluids high speeds are generally excluded. *Mobile liquids*, such as water, form a second section, often referred to simply as hydromechanics. Aerodynamics deals with *gaseous fluids*, such as air, which have very little resistance to deformation and in which high speeds can be attained with comparative ease. Such fluids, however, exhibit considerable changes in density whenever the speed is high or the depth of the fluid is very great, and thus aerodynamics is conveniently subdivided into two related studies, namely that part which deals with *incompressible fluids* in which changes of density can be ignored, and that dealing with *compressible fluids* in which density changes must be taken into account. Until recently the speed of aircraft was such that the air could be considered as incompressible without appreciable error, but to-day we must reckon seriously with the changes in density which occur around a machine in flight.

THE PROGRESS OF FLYING FROM THE EIGHTEENTH CENTURY ONWARDS

Although the purpose of this book is an account of the science which lies behind practical aeronautics, it is as well to give

here a brief sketch of how mechanical flight was finally achieved. In doing so we omit those devices, such as balloons and airships, which rise from the ground because of their buoyancy and consider only machines which are heavier than air.

The possibility of mechanical flight in this restricted sense turns upon the fact that a body moving through the atmosphere experiences a force due to the resistance of the air. This force is conveniently divided into two components: one, called *lift*, usually directed vertically and opposed to the weight of the body and the other, called *drag*, acting in a direction opposed to that of the motion. In the practical problem of flying lift is clearly desirable, whereas drag represents an unavoidable loss of energy. Certain bodies, classified generally as *aerofoils*, are capable of producing much more lift than drag, and since lift in general increases with the speed of the air over the aerofoil surface, it is clear that if an airstream can be made to flow sufficiently rapidly past the aerofoil, a force greater than the weight and acting in the opposite direction can be produced and the body made to rise and sustain itself in the air. To cause the air to move rapidly over the aerofoil an *airscrew* (or *propeller*) is generally used, and the problem of designing a conventional aircraft is therefore essentially that of assembling lift-producing and weight-carrying bodies of low drag around an airscrew and engine.

This scheme, although simple, is far from intuitive and has no exact counterpart in the natural flight of birds. Early would-be aviators, as might be expected, were obsessed by the idea of producing something closely resembling a bird, and the idea of the fixed wing was not clearly formulated until the early part of the nineteenth century, when Sir George Cayley carried out a remarkable series of experiments with model aircraft. Cayley, more than any man of his time, really understood the essential requirements of mechanical flight. He also realized the necessity of making aircraft stable and invented

the principle of the dihedral or the arrangement of the wings in a flat vee, a device still used in all sorts of aircraft from the model glider to the full-size machine. It is possible that if the light and powerful internal combustion engine had been available at the time, Cayley would be honoured to-day as the first man to achieve mechanical flight.

In the nineteenth century considerable attention was also given to motorless flight or gliding, the outstanding name being that of Otto Lilienthal (1846–96) who made many hundreds of successful flights before his death in a crash. Lilienthal clearly realized that to achieve mechanical flight the aerofoil supporting surfaces, or wings, must be designed to produce a high lift/drag ratio, and he demonstrated by experiments that a slightly cambered or arched surface is considerably better, judged by this criterion, than a flat wing of the same dimensions and moving at the same speed. In his flights Lilienthal, who believed that one should 'learn to fly' in the sense in which one learns to ride a bicycle, achieved a measure of control by shifting his weight relative to the wings; the idea of modifying the geometry of the aerofoil to change the lift had not yet arrived, nor was the importance of stability in flight fully appreciated.

Powered flight was studied extensively in America by S. P. Langley, Secretary of the Smithsonian Institution. In an article published in the *Strand Magazine* in 1897 Langley gave an account of his steam driven 'aerodrome' *, a large model which the year before had flown about half a mile over the Potomac river. Langley's account is interesting in two respects among others, first, for the stress placed on the analogy with natural flight (the account is illustrated with drawings of birds' wings and bird skeletons) and secondly,

* The reader should note that in many of the early writings on aeronautics the word 'aerodrome' (literally meaning 'air runner') is used for what we now call 'aircraft' or 'aeroplane'.

for his belief, at this late stage, that Newton's calculations seem to show that mechanical flight is impossible. Undoubtedly the reference here is to what is known as Newton's 'Sine-Squared Law', a formula which indicated values of the lift far inferior to those which are found in practice, and for which Newton has sometimes been blamed, quite unjustly, for having delayed the invention of flying by at least half a century. Actually the blame lay more with those who applied Newton's theory to the atmosphere; the great scholar had postulated a rarefied and frictionless hypothetical medium, and not air, in enunciating his result.

True mechanical flight was finally achieved on December 17, 1903, when Orville Wright took his seat in a flying machine which he and his brother Wilbur had built and fitted with a small petrol engine. The aircraft 'took-off' with the aid of a car and remained airborne for about 12 seconds. In a later attempt it stayed aloft for 59 seconds and flew a distance of 852 feet.

In considering the Wrights' achievements several features are worthy of note. First, the Wright brothers, although in no sense professional scientists (they were mechanics who owned a small bicycle-repairing shop), approached their problem in what has now come to be recognized as the standard scientific procedure of development from small and simple models to the full-size machine. They were not inventors who had hit on a successful design by chance. Secondly, they introduced at least two features which in a developed form are possessed by all aircraft to-day, namely that of the elevator or horizontal rudder for the control of movement in the vertical, and the principle of the control of rolling by flexing the rear edge of the wings, a duty now performed by the ailerons.

The conventional aircraft of to-day is usually a monoplane, driven by internal combustion piston engines and airscrews. It is controlled for rolling movements by ailerons, or hinged

flaps set in the rear edge of the wings, for movements in the vertical plane by an elevator, or hinged horizontal surface set in the tailplane, and by a rudder, also set in the tailplane, for control of direction in a horizontal plane. With the passage of years the aircraft has lost its angularity and has become streamlined in every sense of the word, but essentially it is still the same machine as that which Cayley saw dimly and the Wright brothers finally built.

On the theoretical side the progress made will be the main subject of this book and will not be anticipated here. It is worth noting, however, that in the first decade of this century F. W. Lanchester made an advance as notable as that of the Wrights, although less spectacular. He laid the foundations of modern aerofoil theory, that is, he found the origin of the all-important lifting power of the fixed wing, but the real value of his work was not recognized at the time. He also attacked the difficult and intricate problem of stability, the property of an aircraft which makes it safe to fly and manœuvre; later Bryan, a distinguished English mathematician, was able to put the whole matter on a sound basis.

At the present time, aircraft are designed by a combination of experiment and theory. The great fundamental problems of fluid motion are still unsolved in the strict sense of the word and there is in aerodynamics much that the real mathematician finds trivial and even repellent. On the other hand, looked at against the background of human achievement and not as an exercise in pure logic, it is an exciting and alluring study. To satisfy the needs of designers and engineers much has had to be introduced which could not be tolerated in the strict discipline of mathematics, but this is a passing phase, and what may have been sheer empiricism – or a flash of intuition – many years ago is now coming into its own as part of the rational development of a subject which is as vital as the air we breathe and in which we fly.

CHAPTER II

The Problem of the Resistance of the Air (I)

WE spend our lives at the bottom of an ocean of air and are therefore only infrequently conscious of its resistance to motion. To most of us air resistance becomes a ponderable entity only when riding a bicycle against a moderate or high wind, for in these circumstances we have to use our muscles to overcome the considerable amount of extra resistance engendered by the higher relative speed of the air stream past the bicycle. Although we invariably speak of the strength of the wind as a *speed*, we are almost always made aware of it as a *pressure* caused by the resistance set up by a solid body on which the air current impinges. In all problems of fluid motion it is, of course, immaterial whether we consider the body at rest, with the air flowing past, or the body moving through still air, provided that the relative velocity is the same in both cases.

The physical fact that all bodies placed in a current of air experience a net force, or resistance, is so commonplace that it is universally accepted without comment. Yet this simple fact plays an exceedingly important part in our lives. Because of air resistance seeds and pollen are scattered, gales uproot trees, and birds, insects and aircraft fly. It is a curious fact that despite the universality and importance of air resistance, accurate measurement of its magnitude, and scientific knowledge of its origin and features, came considerably later in history than similar knowledge of the distances and movements of heavenly bodies. For the earliest systematic and reliable data on air resistance we must go back to the invention

of firearms, and it was the urge to provide better weapons of war which produced as a by-product much of our knowledge of this subject.

AIR RESISTANCE IN BALLISTICS

When ballistics, the science of gunnery, began to be seriously considered it was first thought that the resistance of the air was very small compared with the force of gravity. It was not until the time of Benjamin Robins (1707–56) that the real importance of the air resistance term began to be appreciated. In 1687 Newton had carried out experiments to determine the value of the resistance force by dropping spheres of various weights and diameters from the dome of St Paul's Cathedral (*Principia*, II, 8) and had found in this way support for a law he had deduced theoretically, namely that the resistance is proportional to the square of the diameter of the sphere and to the square of the velocity. Newton's experiments were, however, necessarily confined to rather low velocities. Robins made measurements at much greater speeds by the device known as the ballistic pendulum and obtained in this way some valid experimental results which are best summarized in his own words: 'The theory of the resistance of the air, established in slow motions by Sir Isaac Newton and confirmed by many experiments, is altogether erroneous when applied to the swifter motions of musket or cannon shot; for that, in these cases, the resisting power of the medium is augmented to near three times the quantity assigned by that theory; however, this increased power of resistance diminishes as the velocity of the resisted body diminishes, till at length, when the motion is sufficiently abated, the actual resistance coincides with that supposed in the theory' (*New Principles of Gunnery*, by Benjamin Robins, F.R.S.).

Thus, in the eighteenth century, ballisticians formed what

we know to be a reasonably accurate picture of the way in which the resistance force varies with velocity. Robins' experiments covered speeds in excess of 1,200 feet per second (about 820 miles per hour), that is, above the velocity of sound waves in air, a very significant fact as will be seen later. Moreover, it was soon discovered that at these high velocities the resistance suddenly jumps to very large values, a fact which was attributed, quite correctly, to the elasticity of the air. However, at the end of the eighteenth century the true implications of this enhanced resistance were not properly comprehended. Parkinson's *System of Natural Philosophy* (1785), a treatise on what we now call mathematical physics, discusses the matter in the following terms: 'The increase of resistance depends upon the degree of compression [of the air] produced by the body and encreases so fast as to render all attempts to augment the velocity of military projectiles, beyond a certain point, very inconvenient or perhaps impracticable. The velocity may be so great as to produce a degree of compression that destroys the body's motion and it will then be reflected in an opposite direction. Thus small shot are often ineffectual, or repelled, when fired with very great charges of powder.'

It is perhaps hardly necessary to mention that the disconcerting phenomenon described in the final sentences does not really occur. There is no such theoretical upper limit set by resistance to the velocity which can be attained in air, except that if the speed be high enough a body will eventually burn away or vaporize, as a meteorite does, because of the intense heat created by its motion through the resisting medium. What is true, as will be shown later, is that there is a certain band of velocities to surpass which demands a very large increase in power compared with that required at lower speeds, but the resistance of the air implies nothing in the nature of an absolute barrier to motion.

In gunnery the resistance of the air is all-important. If gravity were the only force to be considered a modern field gun, throwing a shell with, say, an initial velocity of 1,700 feet per second at an elevation of 30° to the horizontal, would have a range of about 26,000 yards. The same shell fired with the same muzzle velocity and at the same elevation travels only about 12,000 yards in air, so that in this case the presence of the atmosphere reduces the range by more than half. The enormous range achieved by the giant V2 rocket was in no small part due to the fact that most of its path was at heights at which the air is so thin that resistance is only a small fraction of that encountered near the ground. If air resistance could be ignored, or expressed by a single simple mathematical formula, the calculation of accurate gunnery range-tables would be an easy matter instead of the involved and laborious process which it now is.

Thus before flying was possible the need for an accurate determination of air resistance made itself felt, and the ballisticians of the eighteenth and nineteenth centuries made considerable contributions to its experimental determination. About the same time the classical hydrodynamics was being greatly developed, but its contribution to the problem of fluid resistance, although quite explicit and easily comprehended, could hardly be accounted a success. To the multitude of real problems involving the resistance of fluids, from the pressure-drop in a pipe to the motion of cannon balls through the air, the mathematical science returned the same answer, that the resistance was exactly zero. The complete failure of fluid mechanics at this stage to conform with what was so plainly evident provided one of the major scientific puzzles of the eighteenth and nineteenth centuries and one whose final elucidation did not emerge until the present century.

We shall now consider exactly what this failure was and how it arose, confining our attention at this stage to moderate

speeds, that is, those well below the velocity of sound waves in air (about 760 miles per hour at normal temperatures).

THE THEORY OF FLUID RESISTANCE

Viscosity. Gases and liquids are special cases of the general class of substances known as fluids; air is a mixture of gases which for our present purpose we may consider as one gas. All real fluids have a certain physical property which is known in scientific language as *viscosity.* We commonly speak of certain liquids such as tar, treacle or castor oil as 'thick' and others, such as water, as 'thin'. In lubrication the language is somewhat different and instead we have 'heavy' and 'light' oils. Despite the terminology the property which is expressed by these adjectives has nothing to do with density (which is what a scientist would usually associate with the words 'thick' or 'heavy'); it is that these liquids exhibit in greater or less degree a resistance to change of shape and therefore, if highly viscous, cannot be poured rapidly or easily stirred.

The same property, but at a much lower level, is also possessed by gases such as air but is naturally not as easily detected. The existence of the viscous property can however be described in terms of a simple experiment.

Suppose we have two long wide bands of smooth material mounted close together and arranged so that one can be made to move parallel to the other, the whole apparatus being kept

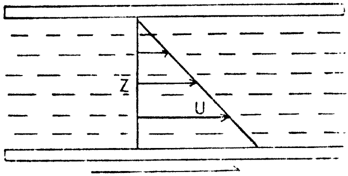

Fig. 1. The Effect of Viscosity in a Fluid.

in a large draught-free chamber (Fig. 1). When the air is at rest we start to move the bottom surface slowly and observe what happens between the upper and lower boundaries. After a time it will be found that this mass of air is also moving (this can be shown by introducing a little smoke into the chamber), and if we measure the velocity of the air at various points between the boundaries we find that it is drifting in a series of parallel planes, with the speed increasing from zero on the fixed boundary to the speed of the moving boundary at the surface of the latter.

This can mean only one thing, that the air is actually sticking to both surfaces and is being dragged along with the moving plane. This motion is in turn communicated to adjacent layers, but with decreasing effect until at the fixed plane the air is at rest again. All of these effects are due to the viscosity of the air.

This experiment is instructive because it illustrates the most important effects of viscosity in gases. These are:

(i) the motion of the boundary is transferred to the air in direct contact with it, and from there to all the air between the boundaries, so that we may speak of the motion which was originally concentrated in the solid boundary being eventually *diffused* throughout the gas just as smoke would be diffused; and

(ii) the speed of the air between the planes changes continuously from one boundary to another. In technical language, a *velocity profile* is set up in the gas, and we call the rate at which the velocity changes between the boundaries the *velocity gradient*. (These terms will occur frequently in the discussions which follow.)

What happens to the air between the boundaries is very like what occurs when a book is placed on a table and the upper cover gently pushed in a direction at right angles to the spine.

The pages slide over each other and we say that the volume is *sheared* and the force which causes this is called a *shearing stress*. In our experiment such a stress exists in the air between the boundaries and is the force which keeps it moving, so that the particles glide over each other in a series of parallel planes. Experiment shows that the shearing stress is proportional to the velocity gradient, or the rate of change of velocity with distance from the boundary. The factor of proportionality between the shearing stress per unit area and the velocity gradient is a convenient measure of the resistance to distortion exhibited by the fluid, and is called the *dynamic viscosity* and denoted by the Greek letter μ (mu). In the simple arrangement considered above the velocity profile is linear, that is, can be represented by a straight line by drawing a graph of velocity against distance from a boundary, but in general such a graph is curved, and if u is the velocity of the air at a distance z from one of the boundaries, the velocity gradient is du/dz in the notation of the calculus. We have then, per unit area,

$$\text{shearing stress} = \mu \frac{du}{dz}.$$

At ordinary pressures the dynamic viscosity depends only on the nature and temperature of the gas. In problems of aerodynamics, however, usually it is not the dynamic viscosity which enters but its ratio to the density of the air ρ (rho). This second measure of viscosity is called the *kinematic viscosity* and is denoted by the Greek letter ν (nu). From its definition,

$$\nu = \mu/\rho.$$

(For air near the ground, ν is about 0·15 cm^2/sec or 0·000156 ft^2/sec in British units.)

The viscosity of the air is a consequence of its molecular structure. We must regard any volume of air as made up of an enormous number of tiny molecules, all darting about at very

high speeds and continually colliding with each other and with the boundaries. This incessant motion, which is quite random, is manifest to our grosser senses as the temperature of the gas, and when we say that the air is at rest we do not mean that the random heat motion has ceased but that the molecules, like a swarm of gnats dancing about in a patch of sunlight on a summer's day remain, as a whole, in the same locality. When in our experiment we move one of the boundaries the effect is to give an additional but unidirectional motion to the molecules of air which strike it and rebound. This additional motion is too slow to interfere appreciably with the random heat motion, but the result is that ultimately all the molecules near the surface acquire a drift in one direction, so that in time the motion which was originally concentrated in the solid boundary spreads throughout the entire gas, but the magnitude of the effect naturally decreases as distance from the source of the additional motion increases. There is a similar effect when a current of air flows over a solid body except that here the boundary acts to oppose the mass motion and the slowing down of the unidirectional motion of molecules near the surface acts as a brake on the drift of the remainder. For this reason the viscosity of a gas is often referred to as its 'internal friction', but the laws of such fluid friction are very different from those of solid friction such as one finds, for example, with the brakes of a car.

The Classical Hydrodynamics and the Problem of Resistance. We can now examine in detail what the hydrodynamics of Euler and Bernoulli and their successors has to say about fluid resistance. In its original form the classical hydrodynamics dealt only with the motion of an *inviscid* or *ideal fluid*, defined as one which is completely free from internal friction. Such a fluid is, of course, a mathematical abstraction, but so are 'rigid' bodies or 'perfect' insulators, and it might be argued that the viscosity of air is in any case so small that the

results obtained by taking it to be absolutely zero would not be far removed from the truth. We therefore investigate the results of this hypothesis by considering in detail the problem of a body of simple shape, namely a long circular cylinder, completely immersed in a stream of an ideal fluid.

The problem may be considered in two parts: first we calculate the *velocity field* or the way in which the speed of the stream changes as it meets the body, and secondly, from the velocity field we deduce the *pressure field* around the body and hence the system of forces. Our object is to determine the resultant force which the stream exerts on the cylinder.

Streamlines. Hydrodynamics makes considerable use of the

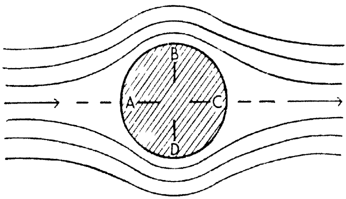

Fig. 2. Flow of an ideal fluid past a circular cylinder.

concept of streamlines, or lines imagined drawn in the fluid to indicate the direction of flow at any point. A streamline is defined as a curve which is always tangential to the flow, so that fluid cannot cross a streamline but only flow along it, and we can thus imagine adjacent streamlines to form a series of tubes through which the fluid is passing. The picture of flow given by the pattern of streamlines is, however, much more than a chart of flow direction – it is at the same time a map of the velocity field and one that is quite easily read because of the simple rule that where the streamlines are close together the velocity is high, and where they are widely separated the fluid is moving slowly. This is exactly what would be expected if the streamlines indicated the position of real tubes because,

in a fluid of constant density, wherever the tube narrows the velocity must increase if the same mass of fluid is to pass in a given time.

When the streamlines retain the same shape at all times the flow is said to be *steady*. For a long circular cylinder placed across the direction of flow of a uniform stream of an ideal fluid the streamlines are easily calculated for steady motion. When the curves are plotted the result is that shown in Fig. 2, which thus indicates the pattern of flow at any cross-section of the cylinder.

In this diagram it will be seen that at a sufficiently great distance upstream of the obstacle the streamlines are straight parallel lines with equal spacing between them. This arrangement indicates the undisturbed uniform stream. As the cylinder is approached the stream divides symmetrically and the absence of streamlines at A and C shows that at these places the fluid is brought to rest by the body. At B and D the streamlines bunch together, indicating that the flow is swiftest at these points. This therefore is the velocity field in the neighbourhood of the cylinder.

Bernoulli's Theorem. To interpret this field in terms of pressure we need a famous theorem, due to Daniel Bernoulli, which gives the relation between pressure and velocity along a streamline or in a stream tube. Bernoulli's theorem is really a restatement of the principle of conservation of energy, and is usually expressed in the form:

$$p + \tfrac{1}{2}\rho u^2 = \text{constant},$$

where p is the static pressure, ρ is the density of the air and u is the velocity. The quantity $\frac{1}{2}\rho u^2$ is called the *dynamic pressure* of the flow and is the pressure felt on the nose of a body at the point where the impinging stream is actually brought to rest.*

* See Appendix for an account of methods of measuring these pressures.

Bernoulli's theorem may be stated in general terms thus: as the velocity increases, the pressure decreases and vice versa. Stated in this way it has proved a stumbling block to many students, for it is natural to associate high pressure with high winds, but a little reflection will show that this is in no way contrary to Bernoulli's theorem. We feel the pressure of the wind only when we retard its motion, and what Bernoulli's theorem does is to locate the positions of high pressure as the places where the stream of air is blocked and prevented from moving freely. Put in this way it appears as a natural and indeed an inevitable law.

If we replace the streamline or flow map by one showing changes of pressure in the stream we should find the pressure rise as we approach *A* (because the cylinder is compelling the stream to stop, just as a line of policemen, by exerting considerable pressure, hold back a crowd surging forward), then decrease to a minimum at *B* and *D*, where the fluid is moving swiftly and easily, and rise again to the same value as at *A* as we approach the rear of the cylinder (*C*), where the stream is again brought to rest.

The remarkable thing about these diagrams is that if the arrows indicating the direction of flow were removed, it would be impossible to tell which way the stream is moving, because the flow and pressure patterns are completely symmetrical. This means that although the fluid exerts a pressure on the up-stream half of the cylinder (*ABD*), it must also exert an equal pressure, *but in the opposite direction*, on the rear half cylinder (*BCD*). Since the fluid is ideal, that is, has no internal friction, these pressures must make up the entire force exerted by the stream on the cylinder and obviously they exactly cancel, so that according to the classical hydrodynamics a cylinder completely immersed in a uniform steady stream of an ideal fluid would experience no resistance whatever.

The same result, by more subtle arguments, can be proved

for a solid of any shape provided it is totally immersed in an ideal fluid flowing steadily and uniformly. The theorem is known as *D'Alembert's Paradox* and for many years constituted one of the great stumbling-blocks in hydrodynamics. D'Alembert's own words are worth quoting, 'I do not see then, I admit, how one can explain the resistance of fluids by the theory in a satisfactory manner. It seems to me that this theory, dealt with and studied with profound attention, gives, at least in most cases, resistance absolutely zero: a singular paradox which I leave to geometricians to explain' (*Opuscules mathematiques*, 1768). Lord Rayleigh (1842–1919), one of the greatest mathematical physicists of the last generation, summed it up more pithily thus, 'On this theory the screw of a submerged boat would be useless but, on the other hand, its services would not be needed'.

The result is, of course, quite contrary to experience; a stream of real fluid, whether liquid or gas, always exerts a net force on a submerged obstacle. The search for the resolution of the paradox occupied many eminent mathematicians for the next two centuries and led to some of the most important and fruitful developments of the subject.

It must not be imagined from the above that the study of ideal fluids has no practical consequences in aerodynamics, or that the classical hydrodynamics is merely a delight of the mathematician. It will be shown later that in one extremely important branch of aerodynamics, namely that dealing with the lift or sustaining force on a wing, the hydrodynamics of an ideal fluid comes into its own, with its conclusions substantially verified by experiment. The spectacular failure to predict resistance has often been taken as evidence of its essential unreality in all aspects of fluid motion, but as will be seen in the next section, D'Alembert's Paradox is not utterly opposed to experience but rather constitutes a limiting case of a phenomenon of great importance in all problems of aerodynamics.

THE WAKE BEHIND A BODY AND ITS SIGNIFICANCE

The question which immediately arises from D'Alembert's Paradox is this: granted that the mathematics is technically correct (and the reader may be assured that this is the case) does the failure of the theoretical work to conform with experience arise from the neglect of viscosity or from some flaw deep in the fundamental assumptions of hydrodynamics? The answer is that while viscosity or internal friction is not entirely responsible, in the sense that friction itself cannot give rise to forces of sufficient magnitude to explain, for example, how a gale can uproot trees, it is indirectly responsible because without fluid friction we could not get the sort of flow which

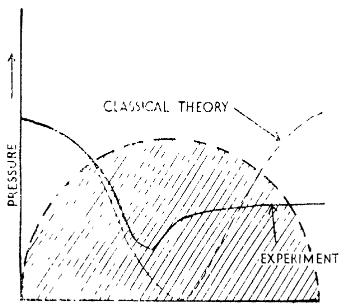

Fig. 3. Distribution of pressure around a cylinder.

actually occurs behind a body where the classical hydrodynamics failed to picture what really happens.

Without viscosity, the resistance force, as we have seen, must arise because of differences in the pressure which the fluid exerts on the body, so that if the resultant force is not to vanish, the pressure distribution must not be symmetrical, as it is in the mathematical solution. A lack of symmetry is, in

fact, exactly what is revealed by accurate measurements of pressure taken over the surface of the cylinder when it is placed in a wind tunnel.* Fig. 3 shows the actual change of pressure over the surface compared with the theoretical rise and fall. Over the front half cylinder, theory and practice are in very fair agreement, but at the rear the theory does not even approximately conform with reality. The expected rise of pressure does not take place, so that the force on the up-stream half cylinder is not balanced by the force on the rear half, and the body therefore experiences a net resisting force, or *drag* as it is termed in aerodynamics.

The origin of the resistance force in this case is therefore to be found in the fact that with a real fluid the flow behind the body is quite unlike that over the fore part. The difference is clearly shown when smoke is introduced into the air stream, thus making the pattern of flow visible. Over the up-stream half cylinder the flow divides very much as indicated by the hydrodynamical theory but, on reaching the downstream half, it leaves the body and forms a clearly defined *wake* or region of disturbed flow behind the obstacle. The air stream has found it difficult to turn the corner and the flow has failed to conform with the shape of the rear half of the cylinder. In technical language the flow *separates*. We shall see later that separation is of immense importance in all problems of aerodynamics.

THE EFFECT OF VISCOSITY ON RESISTANCE

So far we have not introduced viscosity directly into the argument, the resistance described in the preceding paragraph

* A wind tunnel is an indispensable adjunct of experimental aerodynamics, and consists essentially of a large pipe (of circular or rectangular cross section) fitted with special devices to ensure that the stream of air through the working section is as smooth and as uniform as possible. See Appendix.

being attributed essentially to the shape of the body and thus to the geometry of the flow. Viscosity enters directly in quite another way, owing to the fact that a viscous fluid, whether liquid or gas, always adheres to the surface of a body, that is, there is no relative motion, or slip, at the places where body and fluid meet.

Mathematical Theory. At this point it is necessary to interpolate a few remarks on the mathematical technique required in dealing with problems of fluid motion. Basically we use two fundamental laws: one is that of the conservation of mass, which in our case says that fluid is neither created nor destroyed during the motion, and the other is Newton's Second Law of Motion, that the force required to change the speed of any element of the fluid is measured by the product of the mass of the fluid element and the acceleration produced. The first of these leads to what is known as the *equation of continuity*, which acts as a kind of censor on the mathematical formulæ, ensuring that nothing which violates the principle of the conservation of mass can enter into the final expressions. The Second Law yields what are known as *Euler's Equations of Motion* (after the great Swiss mathematician by whom they were first enunciated). As we have said, they are really Newton's equations of ordinary dynamics, but since a fluid in general not only changes its velocity from point to point in space at any one instant but also from moment to moment of time at any one point, the components of the velocity resolved along the three co-ordinate axes must be considered as functions both of space and time. Thus each component of acceleration is made up of four terms involving the partial differential coefficients or rates of change of the velocities with respect to the three spatial co-ordinates and the time. This represents a very considerable complication compared with, say, the dynamics of a particle.

Having written down the acceleration of an element of the

fluid and brought in the mass by multiplying it by the density we must now equate this product to the forces. These may be of two kinds: external forces, such as gravity, which for the moment we may ignore, and internal forces due to pressure. The ancients said that Nature abhors a vacuum, but they had much more truly said that Nature loves uniformity above all else and if there are inequalities of pressure the fluid will move to smooth them out. The driving force in this case is the *pressure gradient* or rate of change of pressure with distance. We meet examples of this every day; when the meteorologist announces that a depression, or area of low pressure, is located over the North Atlantic and that winds will soon increase to gale force over the country, he means that a pressure gradient exists over this part of the globe and that as a consequence air is rushing from regions of high pressure to those of low pressure, except that in this case the air cannot proceed directly towards the centre of low pressure, but is forced on to a curved path because of the rotation of the earth. The wind speed can then be calculated with considerable accuracy by using Euler's equations to express the fact that the air adjusts its speed to achieve a balance between the pressure gradient and the forces which arise because it is moving along a curved path on a rotating earth.

Thus each of the three equations of Euler for an ideal fluid consists of an acceleration group containing four terms equated to the pressure gradient and the external forces. The extension of these equations to viscous fluids was made independently by the French mathematician M. Navier (1785–1836) and the English mathematical physicist Sir George Gabriel Stokes (1819–1903). The addition consists in all of nine terms, containing the coefficient of viscosity and second order partial differential coefficients of the component velocities, so that we have in the *Navier-Stokes equations* four distinct groups of forces to balance:

those depending on the density, and on space and time variations of the velocities (Euler).	those depending on the viscosity, and on space variations of the velocities (Navier and and Stokes).
those depending on the pressure gradients.	those arising from external agencies.

The problem is to find the velocities and pressures which will achieve the balance and at the same time to satisfy the equation of continuity.

The terms introduced by Euler arise solely from the fact that a fluid has mass or density, while those added by Navier and Stokes are due to the fact that a fluid is made up of molecules, that is, has internal friction. We call the first group the *inertia terms* and the second the *viscous terms*, and together they describe all the features of a real fluid in motion. The reader should think of a crowd pouring out of a football ground – considered as an entity some of its properties arise because of the sheer weight of people, while others are due to the fact that it is made up of individuals who bump into their fellows and get into each other's way. These two aspects of a crowd represent its inertia and internal friction respectively, and the words are not inappropriate in a more than scientific sense.

The problem of solving these equations, that is, of determining the velocities and pressures, brings us into the realm of higher mathematics. The expressions constitute what are known as second order partial differential equations. Provided that such equations are *linear*, that is, do not involve squares and products of the unknown functions, there exist certain well established methods for their solution, but the problem is, even then, never very easy. Unfortunately, the inertia terms in these equations contain both squares and products of the velocities and the equations are thus *non-linear*. So far, no

mathematical technique has been discovered which can deal adequately with non-linear partial differential equations of the second order and consequently, even at this advanced stage of mathematical knowledge, the equations of motion of a viscous fluid are intractable and hold fast their secrets. The only solutions which have been found are those for certain cases of extremely slow speeds and these have therefore only a limited interest for aerodynamics, although some of the results are of considerable importance in other branches of science.

At such very low speeds we can often afford to disregard the inertia terms, and since these are the terms which contain the squares and products of the velocities, the simplified approximate equations become linear and therefore more manageable. The most famous solution of this type, for purely viscous flow, was discovered by Stokes, and it expresses the resistance experienced by a sphere moving very slowly through a fluid. The result is very simple: if d is the diameter the resistance is $3\pi\mu ud$, where u is the velocity of the sphere and μ is the dynamic viscosity of the fluid through which it is moving.

Stokes' Law, as it is called, is very important in meteorology because it applies to the minute drops of water which make up mists and fogs. Knowing the average size of the drops it is then possible to calculate the maximum speed at which they can fall through the air, and Stokes' Law shows that this speed is extremely small, so small that such drops remain in suspension for very long periods and thus appear to 'float' in the air. They do not float in the true sense of the word because water is 800 times heavier than air; the drops are actually falling, but at an almost imperceptible rate. Stokes' Law indicates that the resistance experienced by a very small sphere equals the weight of the drop as soon as the velocity reaches quite a small value, and when the resistance exactly balances the weight there is no longer any net force to

increase the velocity. The speed at which this balance occurs is called the *terminal velocity* and represents the highest speed which the body can attain when freely falling in air.

Stokes' Law is deduced on the assumption that the inertia terms can be omitted, and once these become of importance, the law is no longer true. Thus ordinary raindrops do not obey Stokes' Law, which would indicate a much higher terminal velocity than is found. To calculate the motion of such large drops exactly is in fact beyond aerodynamics at present, but the problem can be solved approximately.

SKIN FRICTION AND THE BOUNDARY LAYER

At this juncture it appears that aerodynamics has reached an impasse: in nearly all cases of practical interest the inertia terms are too large to be omitted, while to leave out the viscous terms, on the other hand, means a return to the hydrodynamics of ideal fluids which, as we have seen, can

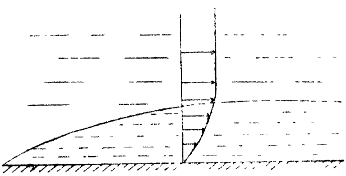

Fig. 4. The laminar boundary layer of a flat plate. (Vertical scale greatly magnified.)

lead only to the sterile result that resistance does not exist. The equations cannot be solved with both sets of terms included and a condition of stalemate is reached.

The difficulty was resolved by the German mathematician Ludwig Prandtl (b. 1875) in his famous paper on the motion of fluids with very small friction, presented to the Third International Congress of Mathematicians at Heidelberg in

1904. It was in this paper that Prandtl gave to the world the words 'boundary layer' (*Grenzschicht*) and thereby introduced a concept which has now become so familiar that it would be difficult to find many papers on aerodynamics in which the term does not occur. The boundary layer theory is a notable example of the way in which physical insight and the study of experimental data can help the mathematician with his peculiar difficulties, by indicating the kind of approximations which make the problems soluble.

Boundary layer theory is easily explained qualitatively, although the detailed mathematical treatment is not simple. Suppose we have a uniform stream of air blowing steadily at a moderate speed over a fixed smooth plate (Fig. 4). With specially designed instruments of small size we can measure the way in which the speed of the air changes as the plate is approached in the direction perpendicular to the flow, that is, we can determine the velocity profile. It is found that the entire change of velocity is confined to a very shallow layer adjacent to the plate and that outside this layer the velocity is uniform. Now the change of the velocity of the air from zero at the plate to that of the undisturbed stream at the top of the layer can be due only to viscosity, and Prandtl made two important deductions: that viscosity has virtually no influence outside the layer, and secondly, that the layer itself is extremely thin.

It is important to realize that the entire effect is confined to an extremely shallow layer, usually only a fraction of an inch thick, so that inside the layer the air experiences a very intense shear. Early in this chapter it was shown that the shearing stress, or frictional force per unit area induced by viscosity, is measured by the product of the coefficient of viscosity and the velocity gradient, and it is the velocity gradient which gives rise to large values of the shearing stress in the boundary layer despite the smallness of the viscosity. This explains how a gas, despite its low viscosity, can produce

significant frictional effects, even when flowing over a flat smooth surface.

Using these facts Prandtl was able to reduce the equations of motion to a form in which they can be solved (usually by laborious but not intrinsically difficult methods), and was thus able to show how to calculate the direct effect of viscosity on the resistance experienced by a body. This sort of resistance is quite unlike that discussed before and has no counterpart in the hydrodynamics of an ideal fluid. It arises because whenever a real fluid flows past a stationary obstacle, or a solid body moves through the air, molecular attraction prevents any relative motion of the fluid and the body at the surface itself. Thus no matter how rapidly air is forced through a pipe or wind tunnel, its speed is exactly zero at the wall, and when an aircraft or a shell rushes through the atmosphere, the velocity of the air immediately adjacent to the surface of the body is, at any instant, exactly equal to that of the moving body, although a fraction of an inch away, outside the boundary layer, it is quite different. We shall describe the flow inside the boundary layer in greater detail in the next chapter; for the present it suffices to point out the general implications of the picture. The field of flow around a body of regular shape (such as an aircraft strut) is thus divided into two regions: an extremely thin layer, covering the surface of the body, where the velocity gradient attains very great values, giving rise to viscous stresses which are too large to be ignored, and the region outside the layer where the pattern of flow is determined by the action of pressure and viscosity may legitimately be neglected.

RESISTANCE AT MODERATE SPEEDS – SUMMARY

We can summarize generally what has emerged so far about the resistance which a body experiences when moving through

the air at low and at moderate speeds (that is, speeds wel below that of sound).

The resistance is made up of two parts:

(i) that which is due to the fact that the flow does no close in around the tail, but separates and forms a wake, and

(ii) that which is due to the fact that the air, because of it viscosity, sticks to the surface of a moving body, and forms a boundary layer.

The first kind of resistance is connected with the shape of the body and is called *form drag*. (It is sometimes called 'pressure drag' because it arises from the difference in pressure on the nose and the tail.) This type of resistance can be reduced or almost entirely eliminated, by carefully shaping the tail so that the flow follows the contour faithfully almost to the point at which the body terminates. An airship, or a fish, is a good example of this sort of shape, which is known in aerodynamics as a *streamlined body*.

The term 'streamlined' has now become popular jargon, bu in aerodynamics it means simply a body which leaves behind it only a very small wake. With such a body D'Alembert's Paradox is very nearly true because pressure differences are almost entirely eliminated, but a resistance is still felt because of the effect of viscosity in the boundary layer. Bodies which have sharply truncated tails (such as a shell) leave behind them a large disorderly wake and have a large form drag such shapes are called *bluff bodies*.

The second kind of resistance, which arises from the intense shear in the boundary layer, is called *skin friction* and can be reduced, but never eliminated, by making the surface of the body very smooth. Skin friction is of minor importance unless the body is streamlined; high polish on the coachwork of a touring car is pleasing but does nothing significant to lower the air resistance of the entire car, which is clearly a

very 'bluff' body. In the case of an aircraft designed for high speeds skin friction may be very important and often more than half the total horsepower of the engine is spent in overcoming it. This leads to a second and better definition of a streamlined body, namely one for which the form drag is much less than the skin friction. To design a body which is to have as low a resistance as possible the first consideration is the reduction of form drag, and this means that great care must be taken to eliminate all protuberances and particularly to give the correct shape to the tail, which at these speeds is rather more important than the head. The next task is to make the whole surface as smooth as possible, thus reducing the skin friction to a minimum.

As an example of what can be achieved in this way let us consider the case of the resistance of a model of an airship envelope, that is, the airship stripped of all excrescences such as fins and gondolas, compared with the resistance set up by a disc of the same diameter held perpendicular to the wind. Such a disc has a high form drag and virtually no skin friction. By carefully shaping the airship model it is possible to reduce its drag to as little as 1/50 of that of the disc. The resistance of the airship model has been brought down until it is almost entirely skin friction, which in this case is only about 2% of the maximum value of the form drag.

In aircraft engineering this process of reducing form drag is known as 'fairing' and is applied to struts, engine nacelles, etc., to reduce the overall resistance. The designer often plots his curves from formulæ to ensure that his shapes have no abrupt changes in curvature to cause separation of the flow, a risk which is always present in a freehand drawing no matter how good it looks to the eye.

CHAPTER III

The Problem of the Resistance of the Air (II)

So far we have not examined closely how air flows past a body, but have been content to describe the motion in broad general terms. The detailed study of air flow near a body, however, brings to light a phenomenon of fundamental importance, whose main features are fairly easily understood although the relevant mathematical analysis is a matter of the greatest difficulty.

TURBULENCE

In 1883 Osborne Reynolds, Professor of Engineering in the University of Manchester, carried out a series of simple

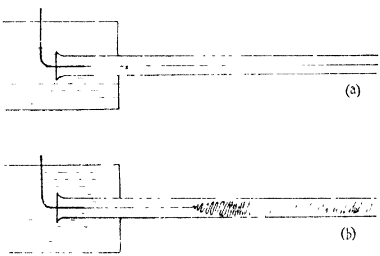

Fig. 5. Reynolds' Experiment (diagrammatic).

experiments whose results he announced to the scientific world in a paper in the *Philosophical Transactions of the Royal Society* with the somewhat lengthy title 'An Experimental Investigation of the Circumstances which Determine whether the Motion of Water Shall be Direct or Sinuous and of the

Law of Resistance in Parallel Channels'. These experiments have made his name immortal in the truest sense of the word.

Reynolds' experiments are very simply described and the apparatus used was essentially that shown in Fig. 5. A long straight glass tube, carefully insulated from external vibrations, is connected to a reservoir, also protected from outside disturbances of any kind. The object of the experiment is to ascertain the nature of the flow through the tube when full, and for this reason a little dye is introduced by a thin subsidiary tube to make the motion visible. The experiment is begun by filling the apparatus, allowing the initial disturbances to subside and then starting a slow flow through the tube. At low speeds the dye makes a thin line parallel to the walls of the tube; this line may sway a little from side to side but essentially it remains unaltered from start to finish and is very little thicker at the outlet end of the pipe than it was at the beginning (Fig. 5 (a)). The experimenter proceeds to increase the speed of flow by small steps, without producing much visible alteration in the appearance of the filament of dye, until quite suddenly a somewhat dramatic change occurs. The coloured line loses its straightness, is violently agitated and the dye quickly spreads over the whole tube, so that at the far end it is no longer possible to distinguish the original filament in the mass of dilute colour (Fig. 5 (b)).

In a general fashion it is not difficult to see what has happened. Before the critical speed is reached the flow is essentially orderly, with the water everywhere moving parallel to the walls of the tube, like a well disciplined regiment on a route march. At the critical speed the motion changes to one of extreme disorder, with the fluid particles still moving in general towards the tube outlet, as before, but now with all sorts of secondary motions causing them to cross and recross the main direction incessantly. The orderly route march has given place to the wild rush of a mob. A

scientist seeing the phenomenon for the first time would almost certainly use the word 'instability' in describing the second type of flow, but it is a very special type of instability with which we are going to deal.

We call the first or orderly type of flow *laminar* and the second type *turbulent*. Laminar motion is sometimes called 'streamline flow', and although this is not a good term it does emphasize, by contrast, the fact that in turbulent flow there is no such thing as a regular pattern of streamlines, the paths taken by individual particles being so complicated and twisted and so different from each other that any precise mathematical formulation is out of the question. All we can reason about are *average values* over a period of time, and throughout this book it must be understood that when we speak of the velocity or the pressure of a turbulent air stream we mean the average speed or the average pressure. To return again to the analogy of the mob, the historian may describe a situation quite accurately by recounting the movements of a mob as a whole, but that is the limit of his information, and he cannot possibly know the exact movements of every man. This state of affairs also holds in turbulence theory; it is improbable, to say the least, that mathematics will ever be able to describe the flow in detail, and even if such information could be obtained, it would almost certainly be too complicated to be used. What mathematical theory can hope ultimately to attain is an accurate description of the flow as a whole, just as a historian, without inquiring into the lives of individuals, traces the story of a nation as an entity.

THE REYNOLDS NUMBER

Reynolds not only demonstrated the existence of the two basic types of flow – laminar and turbulent – but he also found the underlying unifying principle governing the transition from

one type to another and with it (although naturally this was not immediately obvious) what may be called the key to aerodynamics. Experiments show that the transition takes place not only when the velocity (u) exceeds a certain value in a pipe of fixed diameter (d), but also when the velocity is kept constant and the diameter is increased beyond a certain value, and finally, when the product of the diameter and the velocity is kept fixed and the kinematic viscosity (ν) decreased. This means that the transition from laminar to turbulent flow depends on the value of the quantity ud/ν, now universally called the *Reynolds Number.*

The Reynolds number, usually denoted by the symbol Re (or simply R), is the most important quantity in the whole of aerodynamics and therefore deserves closer attention. It is what mathematicians call a *non-dimensional quantity* or a *pure number.* In physical science certain quantities, the most obvious of which are mass, length and time, are regarded as fundamental since, like the elements in geometry (point, line, etc.) they are independent of each other and cannot be defined in terms of yet simpler quantities. All other physical quantities are built up of these elements; thus velocity is a length divided by time, density is a mass divided by a volume, that is, by the cube of a length, kinematic viscosity is an area, or the square of a length, divided by time, and so on. Such composite quantities are said to have *dimensions.* To measure these quantities we employ systems of *units,* such as feet or metres for length, pounds or grams for mass, seconds for time and so on. It is an unfortunate fact that more than one system of units is employed, so that the same velocity, for example, is represented by different numbers in different systems, and mathematics deals exclusively with numbers. Thus although we have grown accustomed to expressing physical quantities by numbers, it must be remembered that these are unlike the pure numbers which make up mathematics; the number π always

has exactly the same value no matter whether we are measuring the area of a circle in square feet or square metres, whereas the number indicating the speed of sound in air at a certain temperature may either be 1,120 when expressed in feet per second or 341·4 in metres per second. The numbers used in calculation in applied mathematics are, in fact, ratios of some measurable quantity with an arbitrary fixed standard, and the ratio depends on the system of units used, so that throughout a calculation we must use one set of units and one only. Which one we use is a matter of convenience or taste.

A little reflection will show that the mathematical expression of a law of nature must not depend in any way upon the system of units used. Thus when Newton enunciated his universal law of gravitation, that the attraction between two bodies is proportional to the product of their masses divided by the square of the distance separating them, he was stating something whose truth or falsity does not turn in any way on whether the masses are expressed in pounds or grams or the distance in miles or metres. Pounds and feet are defined by Act of Parliament, and to dispute the proposition that a genuine natural law must always be capable of being expressed in a form independent of all systems of units is tantamount to admitting that the legislature has authority even in the realm of nature. The Reynolds number consists of the product of a velocity and a length divided by a kinematic viscosity. The numerator is therefore made up of a length divided by a time (velocity) multiplied by a length (diameter), thus making a $(\text{length})^2 \div (\text{time})$ while the denominator, being a kinematic viscosity, is also a $(\text{length})^2 \div (\text{time})$. That is, ud and ν have the same dimensions and so their quotient ud/ν is non-dimensional. Hence, provided we are consistent in our units for both numerator and denominator, the Reynolds number is the same no matter whether the measurements are made in the British, the metric or any other system of units. (Obvi-

ously we must not mix systems; if we start off by measuring u in feet per second we must measure d in feet and ν in square feet per second, but having observed this rule we may forget the units employed.)

Reynolds' result is sometimes stated thus: flow in a straight pipe becomes turbulent if *Re* exceeds 2,200 and is laminar if *Re* falls below 2,200. Actually, the true result is not quite as simple as this, for later experiments have shown that by carefully shaping the entry to the pipe and taking other precautions against accidental disturbances it is possible to maintain laminar flow for *Re* well above 2,200, but in these circumstances even a slight disturbance will cause the transition to take place. On the other hand it seems to be true that turbulence cannot persist if *Re* falls below 2,200, no matter how irregular the flow may have been initially. These refinements need not concern us unduly at present; what is important is to realize that in demonstrating that the existence of the two basic types of flow turns upon the pure number ud/ν, Reynolds had discovered something fundamental in fluid motion. How fundamental it is has become increasingly evident with the development of aerodynamics, and without the Reynolds number it is doubtful if practical aerodynamics could have proceeded at all; it would have been choked by the mass of accumulated data.

THE NATURE OF TURBULENCE

Although there is no exact definition of turbulence the state is easily recognized, and in fact nearly all 'natural' motion is turbulent. Both laminar and turbulent motion are seen, for example, in the smoke from a cigarette held steadily in the hand. Near the lighted end the smoke rises in a thin steady line, but higher up the stream of hot air loses direction and exhibits all the characteristic features of turbulence.

Turbulence is exceedingly important in meteorology, especially in the layers of air near the ground. Normally, the surface wind is very turbulent, that is, it consists of a rapid succession of gusts and lulls, and this aspect of the motion of the air has much to do with the shape of life as we know it. We have seen that one of the effects of viscosity is to cause diffusion of motion throughout a fluid, and that this property can be traced to the effects of the irregular and unceasing agitation of the molecules. The same agitation also causes the diffusion of heat (or, as we commonly call it, the conduction of heat) in a fluid and also the diffusion of matter, but because of the small size of the molecules and the extremely short distances they travel before hitting other molecules, molecular conduction and diffusion in gases are rather slow processes. Turbulence provides a kind of magnified viscosity, conductivity and diffusivity in which wandering masses of fluid, called eddies, carry motion, heat and foreign matter from one part of the fluid to another. To understand the effects of turbulence one must think of it essentially as a continuous mixing process in which the eddies behave like molecules.

Working on this analogy, meteorologists (principally the British mathematicians, Sir Geoffrey Taylor, L. F. Richardson and Sir David Brunt and the Austrian meteorologist Wilhelm Schmidt) have made considerable strides in the study of atmospheric turbulence and we now know that the atmosphere possesses an enhanced viscosity, conductivity and diffusivity due to turbulence. It has been found that in the atmosphere the coefficients of eddy viscosity, eddy conductivity and eddy diffusivity are about a hundred thousand times larger than the corresponding molecular coefficients, and in this way the air near the ground is provided with very efficient automatic mixing. If this were not so, life as we know it could not proceed. The air at breathing level would either be intolerably hot or very cold and either extremely humid or very dry;

there would be little in the way of airborne pollenation and the widespread scattering of seeds, and smoke would cling to the ground, probably for days. It would be a climate of extremes, utterly unlike what we know now, for turbulence is the great atmospheric diffusing agency, controlling such apparently diverse processes as the evaporation of water from seas and lakes, the spread of heat from the ground to the upper air and the cleansing of the atmosphere from local pollution.

Although the motion of the air constituting the first few hundred feet of the atmosphere is extremely irregular, with oscillations having periods varying from a fraction of a second to many minutes, this feature tends to die out as we ascend and at heights in excess of about 2,000 feet the frictional drag of the earth's surface is no longer appreciable. At such levels the wind is almost non-turbulent, in the sense that rapid oscillations of the type found near the ground can no longer be detected even by sensitive instruments. Thus an aircraft normally flies in a non-turbulent air stream, but this statement needs rather careful qualification. Although the rapid and incessant turbulence which characterizes flow near the ground has disappeared, large-scale oscillations still appear, but at relatively long intervals. In certain meteorological conditions the air can be 'bumpy' up to great heights and evidence is now accumulating to show that vertical currents can exist of magnitude sufficient to make the problem of the design of very large aircraft a difficult matter. The study of turbulence in the free atmosphere (that is, at heights well above the zone of the earth's frictional drag) is still in its infancy, mainly owing to the difficulty of making reliable measurements of the magnitude and extent of vertical gusts at great heights, but the question is one which must be thoroughly investigated if aircraft continue to grow in size.

CIRCULATION, VORTICITY AND THE FORMATION OF EDDIES

We have spoken above of the eddies as the macroscopic counterparts of molecules, but the analogy, indispensable though it is, can be carried too far. Molecules in a gas are permanent bodies and the branch of mathematical physics known as the Kinetic Theory of Gases arose because it was found possible to deduce the physical properties of gases by treating molecules, in the first instance, as if they behaved like very small elastic balls. Eddies are evanescent, formed from the fluid, and they ultimately disappear because they have been absorbed again into the main motion. We cannot treat an eddy as if it were merely a lump of matter with definite mechanical properties, and for this reason the mathematical theory of turbulence is more difficult and less developed than the corresponding molecular theory.

How are eddies formed and how does turbulence arise in the first place? It is not difficult to give a qualitative answer to this question, although it constitutes one of the most famous and troublesome problems of mathematics, the solution of which is incomplete to this day. Mathematically, the problem may be expressed thus: suppose we have a fluid in steady laminar motion, in what circumstances will this motion change into turbulent flow? Essentially, this is a question of stability and there is a well-developed mathematical technique for attacking such problems. An expression representing a small disturbance is introduced into the equations, which are then examined to determine the conditions in which the disturbance will subside, indicating that the system is stable, or will grow indefinitely, in which case the system is unstable and the flow will ultimately break down into turbulence. The complexity of the situation is such that the complete solution has evaded the ablest mathematicians for the past fifty years.

Irrotational and Rotational Motion. To form a picture of how eddies originate necessitates the introduction at this point of certain fundamental concepts. We begin by explaining what is meant by irrotational and rotational motion in fluid mechanics. By a well-known theorem in kinematics the movements of a body, however complicated, can always be resolved into two basic types of motion, *translation* and *rotation*, and we can apply the same reasoning to a small volume or element of a fluid. Translation means that the fluid element always

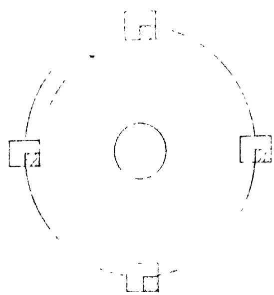

Fig. 6. Irrotational Motion around a Centre.

retains the same angular relation to a fixed set of axes, but the element as a whole may describe any path in space. Rotation means that the element is changing its angular relation to the set of axes or, more simply, that it is turning about its own centre.

We can realize the significance of these terms if we think of the Giant Wheel which provides thrills at fun fairs. This consists of a large wheel around whose circumference cars are hung on hinges, and as the wheel rotates the cars are carried high into the air and down again. If we regard the cars as 'elements' we see that a passenger seated in one of these 'elements' will always remain upright and facing the same

direction, so that although the wheel *as a whole* is rotating about its centre, the motion of the cars is one of translation only (Fig. 6), because they are free to swing on their hinged suspension. If the cars were rigidly fixed to the wheel a passenger strapped to his seat would find himself going through all positions – upright, horizontal and upside down – in a complete revolution of the wheel. The motion of the cars would then be a combination of translation and rotation.*

In applying these concepts to fluid motion we must remember that unlike a rigid body, a mass of fluid in motion allows the elements which compose it freedom to move about independently of the main flow, so that we can speak with precision of the irrotational or rotational flow of small elements only. In Chapter II we discussed the steady flow of an ideal fluid past a stationary circular cylinder. The case dealt with was actually one of irrotational flow, that is, no small element of the fluid possessed rotational motion of its own although the flow as a whole was along curved lines.

Irrotational flow is the simplest type of fluid motion and one which is readily amenable to mathematical treatment. It is the flow which is found outside the boundary layer when a non-turbulent air stream moves past a body. In such a flow the motion of the elements is entirely translational; when this condition is not satisfied the elements are said to possess *vorticity*, and irrotational flow is often referred to as one in which there is no vorticity. It is also called 'potential flow', a term which arises from the mathematical similarity of the theory of this type of motion to that used in certain other branches of mathematics, notably electricity and gravitation.

Circulation. Suppose now we make our cylinder rotate at a steady rate in a fluid at rest. The fluid adheres to the walls of the cylinder and the innermost layers are carried around with

* Another familiar example of circular motion with rotation is that of the moon and the earth, with the moon as the 'element'.

it. Provided certain precautions are taken to avoid turbulence, the flow set up by the rotating cylinder will ultimately be steady and irrotational, and it can be proved by fairly simple arguments that in this case the streamlines are concentric circles over any one of which the velocity is constant, while the velocity as a whole decreases inversely as the distance from the centre (Fig. 7). If r is distance from the centre of the

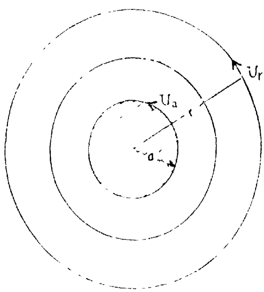

Fig 7. Flow around a cylinder with circulation.

cylinder and u_r is the velocity at this distance, we can express this fact by writing

$$u_r = \frac{\Gamma}{2\pi r}$$

where Γ is an unknown constant, the factor 2π being introduced for reasons which will become plain later. It follows that

$$\Gamma = 2\pi r u_r,$$

and we say that Γ is the *circulation*, which in this simple case is equal to the product of the circumference ($2\pi r$) of the circle and the velocity (u_r). Since u_r is inversely proportional to r it is obvious that Γ has the same value for all the concentric circles, and if the cylinder is of radius a, in particular

$$\Gamma = 2\pi a u_a,$$

where u_a is the speed of the fluid touching the surface of the cylinder and is therefore equal to the speed at which the cylinder rotates.

Circulation is a fundamental quantity in aerodynamics, as we shall see later. When the velocity is no longer constant and the closed curve is not a circle a more complicated definition is necessary, but this definition is built on the same lines as the one given above. We can always think of circulation quite simply, if not absolutely accurately, as the length of the circuit multiplied by the average value of the velocity over it.*

Returning to our circular cylinder we have now discussed two cases: (i) streaming flow past the cylinder, with no circulation, and (ii) flow around the cylinder with circulation but no general streaming flow. The combination of these two types of flow gives rise to a phenomenon of great practical and theoretical interest, but we shall postpone consideration of this until Chapter IV, when we deal with the problem of lift.

Vorticity. So far we have considered circulation about a body of finite size only; if we let the cylinder shrink indefinitely we reach the mathematical definition of the vorticity of the resulting infinitesimal element as the ratio of the circulation around it to its area. This simply means that the vorticity of an element is a quantity of the same nature as angular velocity. Stokes explained the idea of vorticity by imagining that a small element of the moving fluid suddenly becomes solid; if it then spins like a top the original motion would have possessed vorticity, which could be measured by the angular velocity of the solid particle. This is perhaps the easiest way of all of understanding what the word 'vorticity' means in fluid mechanics.

* For the mathematically minded reader: the exact definition of circulation is 'the line integral of the tangential component of the velocity along the curve'.

There is a famous theorem due to Lord Kelvin (Sir William Thomson) which says 'In an ideal fluid circulation and vorticity can neither be generated nor destroyed'. This means, in effect, that in the ideal fluid theory once a motion is irrotational it stays irrotational or, in other words, any motion which can be started from rest in an ideal fluid must be forever devoid of vorticity. This might be thought sufficient to discourage anyone from taking much more interest in the matter, but, fortunately for aerodynamics, mathematicians, following the lead of Helmholtz, have been content to study in great detail motions having circulation or vorticity in ideal fluids without bothering how they could possibly arise. At the time these investigations, although elegant, must have seemed particularly 'useless' (except as a source of questions for examination papers), but later developments were to show that they are of fundamental importance in aerodynamics.

We now know that vorticity can be generated by the action of viscosity, and since in flow past a solid body the effect of viscosity is virtually confined to the boundary layer, we see that vorticity will be found, if anywhere, in this layer and in the remains of the layer in the wake. Owing to the intense shear an element of air caught in the boundary layer is both rotated and distorted before being flung into the wake.

Vorticity has a marked tendency to be concentrated into small regions of the fluid, either as isolated patches, tubes or sheets. The isolated patches are usually called eddies (or vortices), and when a body such as an aircraft or a shell moves through air originally free from rotation (such as the atmosphere above the earth's friction layer), the vorticity generated by its passage will be found only very near the body (in the boundary layer) and in the wake. A bluff body leaves behind it innumerable patches of vorticity, usually the disintegrated remains of its boundary layer, and these account for most of its resistance. To start and launch into the air a family of little

whirlpools means expenditure of energy, which must be drawn from the motion of the body, and thus form drag at moderate speeds is essentially due to vorticity in the wake and therefore, in the last resort, to viscosity.

The eddies in an air stream ultimately die away, their energy dissipated by the same viscosity which gave them birth. Their life history is summed up in L. F. Richardson's parody of Swift's famous lines:

Great whirls have little whirls
That feed on their velocity
And little whirls have lesser whirls
And so on to viscosity.

We have still to explain exactly how vortices form. This is not difficult if we think again of the irrotational flow past the circular cylinder (Fig. 2). As the fluid particles move from near A to B they are gaining speed and at B they have just sufficient momentum, in an ideal fluid, to carry them right on to the rear of the cylinder despite the ever-increasing pressure. With a real fluid, however, the innermost layer sticks to the surface of the cylinder and the diffusing action of viscosity communicates this slowing down to all the other fluid particles. A particle not too far away from the surface has thus to fight its way to the rear against the adverse pressure gradient with the added hindrance of the viscous forces, and it has insufficient energy to do this. It turns back, and the layers of fluid which formed the boundary layer coil up into eddies which ultimately break away and float downstream.

At low speeds it is often possible to see the birth of vortices going on in a perfectly regular fashion. In certain conditions the eddies detach themselves alternately from either edge of an obstacle placed across stream and as they break away they form in the wake the pattern known as the *Kármán vortex street* (after the Austrian mathematician Th. von Kármán, now an American professor, to whom we owe the basic

mathematical analysis of the problem). In this case it is possible to calculate the motion exactly, but at higher speeds regularity disappears and the wake becomes a mass of eddies of all shapes and sizes. Turbulence has set in and mob law prevails.

THE EFFECT OF TURBULENCE ON RESISTANCE

Turbulence not only appears in the wake of a body but also in the boundary layer. Fig. 8 shows what happens to the boundary layer if we make the length of the surface sufficiently

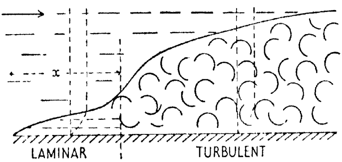

Fig. 8. Laminar and turbulent boundary layer. (Vertical scale greatly magnified.)

great downstream. At first, flow in the boundary layer is entirely laminar, but at a certain point the layer suddenly thickens and starts to grow more rapidly. The distance downstream, x, at which this occurs indicates the ***transition point*** and is determined by the fact that flow in the boundary layer becomes turbulent if the Reynolds number, ux/ν, exceeds a certain critical value, so that turbulence can appear in a boundary layer if either the velocity, or the length of the surface downstream, is large enough. When this occurs the situation becomes more complicated. The effect of the turbulence is to mix up the slow and fast moving fluid elements very thoroughly, with the result that the velocity becomes much more uniform over the greater part of the depth of the layer, but immediately in contact with the surface there is a very thin ***laminar sub-layer*** in which the velocity gradient is extremely

high (Fig. 9). This sub-layer is important because it determines whether a surface is 'rough' or 'smooth' in the aerodynamical sense. If the surface irregularities (such as rivet heads) are small enough to be entirely submerged in the laminar sub-layer they have virtually no effect on the total resistance, and the surface is *aerodynamically smooth*, but if the irregularities are large enough to protrude through the sub-layer into the turbulent layer above they cause a noticeable increase in the total skin friction, and the surface is said to be *aerodynamically rough*. In practice, the fact that the laminar

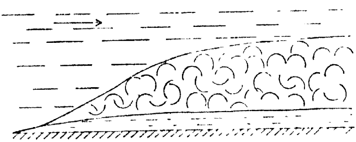

Fig. 9. The Laminar sub-layer. (Vertical scale greatly magnified.)

sub-layer is very shallow means that a surface which is aerodynamically smooth at high speeds must also be very smooth in the ordinary sense of the word, but there would be little point in giving it an 'optical finish' in an attempt to get a further substantial reduction in skin friction.

The effect of roughening the surface is generally to increase the skin friction and consequently the total resistance, but this is not always so with a bluff body such as a sphere or cylinder. We have already seen that in these cases the form drag is high because the flow separates and forms a broad wake in the rear, and it is obvious that if a means could be found of delaying the separation the result would be a smaller wake and a lower total resistance. Now the motion of the air near the surface of such a body is a struggle of the forward pull of the free stream at the upper limit of the boundary layer against the retarding influences of the boundary and the pressure gradient. If

the motion is entirely laminar, the air in the boundary layer can be kept moving towards the rear only by momentum fed in from the free stream by the diffusing action of viscosity, but if the boundary layer flow becomes turbulent there is, in addition, a powerful mixing effect, which means that reinforcements of fast moving air are brought in from outside to replace those masses of air which have been slowed down by the combined drag of the boundary and the blocking effect of the pressure gradient. The overall effect is that a turbulent boundary layer gets much further along the surface before it separates, with the result that the loss of pressure at the rear is restricted to a much smaller surface and the form drag is diminished.

This explains a remarkable feature of resistance law for bluff bodies, first noticed in 1912 by Constanzi for spheres in water and by Eiffel for spheres in air, that in the region of Reynolds numbers between about 200,000 and 500,000* the resistance suddenly *decreases* with increasing velocity over a short range of velocities before starting its normal increase again. The 'kink' in the resistance curve occurs when the speed of the main stream becomes sufficiently high to make the boundary layer turbulent before separation takes place. This has been shown experimentally by Prandtl, who used smoke to demonstrate the wide wake obtained with a smooth sphere and then introduced artificial turbulence in the boundary layer by fixing a wire hoop around the sphere, thus making the surface rough. Photographs of the flow then showed a much smaller wake and with this arrangement reduced resistances were obtained even at fairly low Reynolds numbers. The phenomenon is of great importance in the design of shapes of low resistance,

* In most cases of practical interest in aerodynamics the Reynolds numbers are very large. A ball of diameter 3 inches travelling at 30 miles an hour through the air has a Reynolds number of nearly 80,000, while full size aircraft operate at Reynolds numbers running into many millions.

and a body which is well streamlined at large Reynolds numbers, when the boundary layer flow is turbulent, may give a disappointing performance at lower Reynolds numbers when the boundary layer is laminar. It is a well-known fact that a golf ball which has a rough or dimpled surface can be driven farther than one with a smooth cover (a fact said to have been discovered in the early days of golf by Scottish caddies who used damaged and discarded smooth-surfaced balls for practice) and it is possible that some, at least, of the advantage found is due to the phenomenon described above.*

Dimensional Analysis. At this stage it is again necessary to bring in a little mathematics, but of the simplest kind. In analysing complex physical phenomena, such as air resistance, mathematicians find it useful to employ a dissection process known as *dimensional analysis*. We can see what this means by considering a very simple example.

Suppose we wish to find out how the time of the swing of a simple pendulum varies with its length. (To make the problem as simple as possible we disregard such complications as the weight of the string which carries the bob, friction at the pivot and air resistance.) We can start by assuming that the time of a short swing depends upon the mass of the bob, its weight and the length of the string. Now weight (the force with which a body is pulled towards the earth) being fundamentally mass (m) multiplied by the acceleration due to gravity (g), we assume that the required expression is made up of powers only and write as a trial formula

$$\text{time of swing} = \text{number} \times m^a g^b l^c,$$

where l is the length of the string and a, b and c are unknown numbers. We determine these numbers by making the equa-

* A full discussion of the flight of a golf ball would be rather involved. In this connexion it has been pointed out to the author by Mr. L. F. G. Simmons of the National Physical Laboratory that part of the advantage may also be due to the fact that a roughened sphere would be more stable in flight.

tion dimensionally consistent, that is, by ensuring that the right-hand side has the dimension of time and no other. The dimensions of acceleration being (length) ÷ (time)^2, we can do this only by taking $a = 0$, $b = -\frac{1}{2}$, $c = \frac{1}{2}$, so that mass vanishes from the formula and we are left with

$$\text{time of swing} = \text{number} \times g^{-\frac{1}{2}} l^{\frac{1}{2}} = \text{number} \times \sqrt{\frac{l}{g}}.$$

This is as far as the method will take us, and to determine the 'number' or constant of proportionality in the above formula we must either make measurements in a laboratory or solve the problem theoretically. In this case the problem is an easy exercise in elementary dynamics, and the complete solution is the well-known expression

$$\text{time of swing} = 2\pi \sqrt{\frac{l}{g}},$$

that is, the 'number' is 2π. Our experiments, if carefully done, would confirm this.

Rayleigh's Analysis of Aerodynamic Force. The method of dimensional analysis was used by Lord Rayleigh to derive what is perhaps the most important single result in aerodynamics, a universal formula for the force which an airstream exerts on a body. Omitting gravitational forces (such as weight and buoyancy) which can be easily dealt with separately, any force which is produced by the movement of a body of given shape and attitude through the air must depend upon:

(i) the relevant physical properties of the air, in this case its density(ρ) and viscosity (μ),
(ii) the speed of the air relative to the body (u),
(iii) the size of the body.

Since the shape of the body is fixed, any linear dimension (d), such as its length, radius or diameter, can be used as a

measure of its size and the choice is entirely a matter of convenience. The problem then is to find how the quantities enumerated in (i), (ii) and (iii) above must appear so that when grouped together the combination represents a force, that is, a mass multiplied by a length and divided by the square of a time. This cannot be done as completely as in the simple problem of the pendulum and in the end two unknowns – an index and the constant of proportionality – remain, the formula being

$$\text{aerodynamic force} = \text{number} \times \rho^{1-n} d^{2-n} u^{2-n} \mu^{n},$$

in which n is the undetermined power (a pure number). Now $\mu/\rho = \nu$, the kinematic viscosity, so that the above expression can be rearranged as

$$\text{aerodynamic force} = \text{number} \times \rho u^2 d^2 \left(\frac{\nu}{ud}\right)^{n}.$$

In this form some familiar terms can be identified. We know from Bernoulli's theorem that when a stream of air is completely stopped by an obstacle the pressure felt is $\frac{1}{2}\rho u^2$. This is the 'dynamic pressure' which is found at the extreme forward point (the so-called 'stagnation point') of a body moving through air at moderate speed and is therefore a natural constituent of an expression for aerodynamic force. The term d^2 represents any area which is a convenient measure of the size of the moving body (e.g., in the case of a sphere of diameter d this is taken to be the area of the disc which the sphere presents to the fluid, namely $\frac{1}{4}\pi d^2$). The final term is clearly the reciprocal of a power of a Reynolds number and this again indicates the fundamental nature of this quantity, which has emerged naturally and inevitably in the process of dissection. The general expression for aerodynamic force can now be rebuilt with the constituents in their proper form, namely

$$\text{aerodynamic force} = \tfrac{1}{2}\rho u^2 d^2 f(Re)$$

where $f(Re)$ indicates an unknown pure number whose value, for a body of given shape and attitude, depends only on the Reynolds number of the motion. For incompressible motion (which means for speeds well below that of sound) the result can be stated in two equivalent forms: *the aerodynamic force on a body is characterized by a single non-dimensional quantity depending only on the Reynolds number of the motion* or *the stream pattern around a body in flight depends only on the Reynolds number of the motion.*

It would be difficult to over-estimate the importance of this result in theoretical and practical aerodynamics. The ultimate aim of any science is the discovery and enunciation of unifying principles and Rayleigh's formula is a good example of what can be achieved in this way by simple and straightforward mathematics. The theorem not only throws light on the complexities of fluid motion but is an indispensable aid in the presentation of data. It follows that if we have a table of values or a graph of $f(Re)$ for a body of given shape and attitude, the information thus given applies to all bodies of the same shape and attitude, no matter how speed, size or physical properties of the airstream may change.

DRAG COEFFICIENTS

If the force we are interested in is drag we write

$$\text{drag} = \tfrac{1}{2} C_D \rho u^2 S$$

where C_D, called the *drag coefficient*, stands for what we have previously called $f(Re)$ and S is the area referred to above. Knowledge of the value of the drag coefficient of a body means, in effect, that we know completely the force of resistance, so that in the case of a body such as a sphere, which always has the same attitude, all the facts about its resistance can be summarized in a single graph or table showing the

variation of C_D with *Re*. In the case of a body of variable attitude, such as an aircraft wing, a series of such curves or tables is needed, but the enormous economy effected is obvious.

The determination of drag coefficients must be done either by an appeal to theory or by experiment. In the present state of aerodynamics the theoretical determination of C_D is possible only in a few cases of simple shape and low speeds, such as a sphere moving very slowly through a viscous fluid, for which Stokes' well-known solution (Chap. II) is available. In this

$$\text{resistance} = 3\pi\mu u d,$$

and if this is put into the standard form $\frac{1}{2}C_D \rho u^2 S$ with $S = \frac{1}{4}\pi d^2$ it follows at once that

$$C_D = 24/Re.$$

This is one of the very rare cases in which we are able to find the drag coefficient exactly by theory, and the result is true

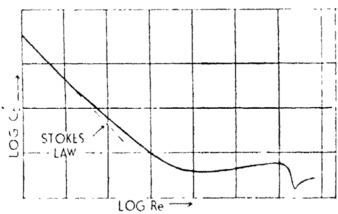

Fig. 10. *The drag coefficient of a sphere.*

only if the Reynolds number *Re* does not exceed unity, which for motion in air means very low velocities or very small spheres, or both. Fig. 10 shows the complete variation of the drag coefficient of a non-rotating sphere moving through a viscous fluid, such as air. The range of Reynolds number covered is very great (from about 0·05 to nearly 4,000,000) so

that a linear scale for Re can hardly be used and the curve is therefore shown 'plotted logarithmically'. That is, instead of plotting values of C_D against Re, to save space log C_D has been plotted against log Re; this means a certain amount of change of shape of the curve compared with that obtained with linear scales, but this does not affect the use of the graph.

The curve shown in Fig. 10 is one of the most famous in fluid motion theory, and is familiar to all students of aerodynamics. It consists of a line drawn through points representing the work of many experimenters in different countries using very different techniques; some of the points on the extreme left of the curve (small Reynolds number) were obtained by measuring the terminal velocities of tiny spheres of amber and wax falling through water, while others represent many years of patient research with larger spheres in wind tunnels in Germany, America and this country. On this curve is to be found evidence of all the features which have emerged from our review of resistance. On the extreme left-hand side, up to $Re = 0{\cdot}5$, the experimental points fall almost exactly on the straight line marked 'Stokes' Law', but from $Re = 1$ onwards the experimental curve moves to the right and thereafter the divergence is too pronounced and systematic to be ascribed to experimental error. This departure indicates the region in which conditions behind the sphere begin to differ widely from those postulated in Stokes' solution (at $Re = 8$ (approximately) it is possible to see the flow leave the surface and form a stationary vortex pattern behind the sphere). Somewhere about $Re = 150$ the vortices detach themselves and float away in the wake to form the vortex street, but the curve of C_D is still smooth and continues to descend. At about $Re = 1{,}000$ the rate of decrease of the drag coefficient becomes progressively less, and from $Re = 20{,}000$ (approximately) to $Re = 300{,}000$ (approximately) there is relatively little change in the value of C_D.

If the drag coefficient were truly constant, that is, were independent of *Re*, we should have a very simple law in which resistance varies exactly as the square of the velocity. A law of this type was deduced by Newton for a hypothetical medium consisting of particles having mass but no length, which are assumed to exert no influence on each other, so that Newton's imaginary fluid has no internal friction. The range of speed in which the drag coefficient is nearly constant is therefore sometimes called the 'Newtonian' or 'quadratic law' region, but the reader is reminded that it is impossible for resistance to vary exactly as the square of the velocity because this would obviously mean that viscosity had no effect, in which case the resistance could only be zero. Nevertheless for many practical applications in which meticulous accuracy is unnecessary the Newtonian law is quite satisfactory – this is so, for example, in ballistics when dealing with simple low-velocity weapons such as mortars. Where Newton's formula goes badly astray is in its prediction of the numerical value of the drag coefficient (and this because once again conditions at the rear of the body are not taken into account), so that in practice Newton's law is followed as regards variation with velocity but with experimental values inserted for the constant of proportionality.

At $Re = 300{,}000$ (approximately) – the exact value depends on the amount of turbulence in the oncoming airstream and this varies considerably from wind tunnel to wind tunnel – the drag coefficient drops extremely rapidly and then recovers, but more slowly. This indicates the critical phenomenon discussed previously when the boundary layer becomes turbulent sufficiently early to delay separation, and the wake narrows.

Fig. 11 shows the variation of the drag coefficient of various shapes for Reynolds numbers lying between 10,000 and 100,000. At the top of the graph is an almost horizontal straight

line which shows the values of C_D for a flat plate held normal to the wind. In this case the Newtonian type of law is very nearly true, a result which might be expected because skin friction is virtually non-existent and form drag is all-important. At the bottom of the graph is the drag coefficient for a well streamlined body, and here again the variation of C_D with *Re* is not very pronounced, but for a different reason. In this case the resistance is nearly all skin friction and in this range of Reynolds numbers the boundary layer becomes turbulent, which means that C_D settles down to a slow variation with Reynolds number (actually, inversely as the fifth root of *Re*). Bodies which are intermediate between these two limiting cases, such as a sphere, have drag coefficients which behave less regularly, chiefly because of the phenomenon of separation.

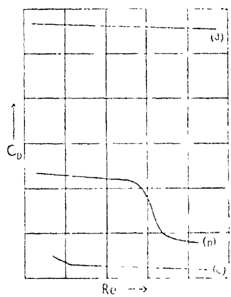

Fig. 11. *Drag coefficients of* (*a*), *disc;* (*b*), *rounded body;* (*c*), *streamlined body at high Reynolds numbers.*

THE PHYSICAL SIGNIFICANCE OF THE REYNOLDS NUMBER

So far we have not considered why the Reynolds number is of such universal importance, and we conclude this chapter by a brief account of its physical significance. A question which always arises in practical aerodynamics is the following: if we have two geometrically similar bodies, such as a full-size aircraft and an accurately scaled-down model, in what conditions will the flow around the bodies be the same? The answer

is clearly that at corresponding points in the two fields of flow the forces acting on a fluid element must have the same ratio to each other at all times. The most important case for aerodynamics is one in which the bodies are totally immersed in the fluid, and gravitational forces, such as weight and buoyancy, are excluded, so that the condition may be simply stated in the form that the ratio between the inertia and frictional forces, acting on corresponding elements, must be identical for the two bodies. It is not difficult to show that this ratio is expressed by the Reynolds number, so, provided that the Reynolds number is the same in both cases, the streamline pattern for the full-size body will be a magnified but otherwise unchanged version of that for the model.

In this sense the Reynolds number expresses the ratio of the inertia forces to the frictional forces in any given situation, and this interpretation throws considerable light on the reasons for the variation of resistance with *Re*. If *Re* is small (less than unity) viscous forces must play a very large part in the determination of the flow pattern; this is so, for example, with Stokes' Law which, as we have seen, is only valid for *Re* less than unity. At the other end of the scale we have Reynolds numbers of the order of millions and it might be thought that this would necessarily imply the dynamics of an inviscid fluid, with its sterile result of zero resistance. The reader should note, however, that the reference velocity in the Reynolds number is the speed of the undisturbed airstream (or, what is the same thing, the speed of the body relative to the undisturbed air) and it is quite true that in the undisturbed stream the air flow is indistinguishable from that of an ideal fluid. Near the body, in the boundary layer, the relative speed varies from zero at the surface itself to the full speed at the top of the layer, so that in this region the quantity ud/ν also varies from zero to its full value, indicating that viscous forces play an important rôle only in the immediate neighbourhood of the

surface. This, of course, is simply a restatement of the boundary layer theory.

Lastly, the reader should note how the type of resistance law changes with the Reynolds number. When *Re* is small, that is, when viscous forces dominate, resistance is proportional to the first power of the velocity, as in Stokes' Law. At higher Reynolds numbers the variation is more nearly represented by

$$\text{resistance} \propto u^{1\cdot75}$$

but the index changes slowly with increasing *Re*, approaching nearer and nearer to the value 2, that is, to the Newtonian law. At very high speeds, namely those in the neighbourhood of the velocity of sound, the resistance law again changes suddenly and completely, but consideration of this extremely important phenomenon must be deferred to a later chapter.

CHAPTER IV

The Theory of the Wing

BEFORE mechanical flight could be achieved, three major problems had to be recognized and solved. These were: the provision of adequate power without excessive weight, the design of efficient supporting surfaces and the attainment of stability in flight. In this chapter we are mainly concerned with the second of these problems.

The solution of the problem of power came with the invention of the internal-combustion engine, but it was some considerable time before it was generally realized that a large source of power is essential for safe flying. Man had always hoped – and even to-day there are those who would still like to think it possible – to emulate the birds and fly by his own muscular power. As early as the seventeenth century Giovanni Alphonso Borelli (1608–79) had declared this to be impossible in his treatise *De Motu Animalium*, from a comparison of the anatomy of man and of birds, but this did not prevent more than one brave but mistaken pioneer finding his death in a pathetic tangle of rods and draperies. Borelli was concerned only with the flapping wing theory, but his conclusion is valid for all forms of mechanical flight.

To our forbears natural flight was a mystery. They realized that there is no counterpart of the balloon in nature and that all flying creatures are heavier than air. Even to-day, when aircraft are commonplace, there are many who find it difficult to comprehend how a medium such as air can apparently support a heavy weight. It is thus not altogether surprising that as late as the nineteenth century the general belief was that nature concealed some deep secret which alone enabled birds

to fly, and that once this secret was uncovered, man also would be able to fly – an attitude not unlike that of the mediæval alchemists to transmutation. A few years before the Wrights' machine climbed slowly into the air at Kitty Hawk, Langley – a typical scientist of his time – wrote 'Nature has made her flying machine in the bird, which is nearly a thousand times as heavy as the air its bulk displaces, and only those who have tried to rival it know how inimitable her work is, for the "way of a bird in the air" remains as wonderful to us as it was to Solomon, and the sight of the bird has constantly held this wonder before men's eyes and in some men's minds and kept the flame of hope from utter extinction, in spite of long disappointment.' Yet the solution, when at last it came, was not that of 'the way of a bird in the air' and the modern aircraft has little in common with nature except the design of the wing.

THE HISTORICAL DEVELOPMENT OF THE AEROFOIL

The full story of the search for the perfect wing which ended in the triumph of the modern aerofoil is too long to be given here; we can at most indicate a few landmarks on the way to success.* The remarkable contributions made by Sir George Cayley (1773–1857) have already been mentioned; coming to the subject when it was a jumble of half truths and vague theories he left it a science. Aeronautics as a whole owes much to Cayley, but aerodynamics in particular emerges as a logical science largely as a result of his clear thinking on fundamentals. There is in the Science Museum a silver disc, the size of a halfpenny, bearing Cayley's initials and the date 1799, on one side of which is a diagram showing the forces on a bird's

* The reader who is interested in the history of flight will find an excellent account in M. J. B. Davy's *Interpretive History of Flight* (Science Museum, 1937).

wing in flight. It is a simple enough diagram – to-day any schoolboy would recognize it as an example of the 'triangle of forces' – but it is significant because in it Cayley clearly distinguishes 'lift' and 'drag'.

Lift was originally called 'useful resistance' and drag, 'resistance proper'. In Chapter I we gave a rough but convenient definition of lift as that part of the aerodynamic force (Cayley's 'resistance of the air') which usually acts in opposition to the weight. This still holds good for level flight, but the precise definition, which is needed in this chapter, is as follows: lift is the component of aerodynamic force acting at right angles to the direction of motion. This means that lift need not invariably act to overcome weight and, during a manœuvre such as looping, may be in any direction.

Cayley not only understood how a flat surface can be made to support a weight in virtue of its forward motion, but he also grasped, in part at least, why an arched or cambered surface, such as a bird's wing, is superior in this respect. His ideas inspired another Englishman, William Samuel Henson (1805–88), to launch in 1842 a grandiose project called 'The Aerial Steam Carriage'. In his Patent Specification Henson describes the action of the wing in the following terms: 'If any light and flat or nearly flat article be projected or thrown edgewise in a slightly inclined position, the same will rise in the air till the force exerted is expended; and it will readily be conceived, that if the article so projected or thrown possessed in itself a continuous power or force ... the article would continue to ascend as long as the forward part of the surface was upwards in respect of the hinder part.'

What Henson is saying here explains how a flat stone can be thrown to skim the surface of a lake, or how an athlete can throw a discus a great distance. It is the elementary principle of the aeroplane, and the drawings which Henson made of the 'aerial steam carriage' bear a strong resemblance to a modern

monoplane, particularly in the method of construction of the wings. Because the whole machine was to be very large (the wing span was to be about 150 feet) and perhaps also because the project was announced in a fashion which shows that our great grandfathers had little to learn about 'publicity methods' (a flight to India was mentioned in the prospectus) the machine in the end was never constructed. This was fortunate for all concerned, for even if the 'aerial carriage' had left the ground it would almost certainly have crashed because of instability. In 1848 Henson, discouraged by failure and financial loss, emigrated to America and appears no more in the story.

The Aeronautical Society of Great Britain was founded in 1866, and at its first meeting Francis Herbert Wenham (1824–1908) read a paper which carried the art of wing design a step farther. He suggested that the major portion of the lift of a wing comes from a relatively narrow strip along the leading edge, so that the most economical shape for a wing is one which is long across the direction of motion and short in depth. This introduces for the first time what is now known as the 'aspect ratio' of a wing, and marks an important advance in wing design.

Wenham also advocated the use of the arched wing, but this feature received fuller examination at the hands of Horatio Phillips (1845–1912) who, after years of patient research, finally evolved a shape which bears a close resemblance to a modern aerofoil, a wing with a thick dipped leading edge, the so-called 'Phillips entry'. The advantages of the curved wing were now becoming apparent to all, and particularly to Otto Lilienthal (1848–96), who, as a pioneer in gliding, needed wings of the highest possible efficiency. In 1889 Lilienthal published his book *Der Vogelflug als Grundlage der Fliegekunst* (The Art of Flying Based on the Flight of Birds) in which he gave his opinion that in the arched

wing 'there lies in all probability the whole secret of the art of flying'.

At this point, the close of the nineteenth century, and just before the appearance of the Wright brothers, it is as well to review the progress made. Apparently it was very little. Cayley had introduced the fixed wing and explained, without recourse to fluid mechanics, the true function of the resistance of the air in sustained flight. Since then a variety of investigations had brought man back to the solution which had been evident in nature from the beginning, that the best shape is that of the wings of large soaring birds. What was clearly lacking at this time was an explanation how the lift force is produced by the motion of the air, why the cambered wing is so much more effective (Cayley and others had divined at least part of the answer here) and last of all how to calculate the lift force and so to proceed to the design of real aircraft.

The final solution was produced (in an undeveloped form) in 1894 by Frederick William Lanchester (1868–1946) to whose 'remarkable physical insight' aerodynamics owes so much. At the time his work was not understood or appreciated, and it was not until many years later that Prandtl gave it the final form which it now bears as the *Lanchester-Prandtl theory*. To comprehend this theory and to understand the magnitude of Lanchester's achievement we must first go back to 1853 and look at a feature of fluid motion studied in the laboratory by a German professor, Magnus, and known by his name, although Newton had recorded the same phenomenon nearly 200 years before in a paper in the *Philosophical Transactions of the Royal Society* for 1672.

THE MAGNUS EFFECT

The existence of a force transverse to the direction of motion, that is, lift, is a commonplace feature of ball games. A 'cut'

tennis ball swerves and a 'sliced' or 'hooked' drive in golf sends the ball sailing into the rough. The deviating force in every case is caused by the spin of the ball, and to understand how this force arises we return once more to the problem of flow around a circular cylinder.

In Chapter III two types of steady flow around a cylinder were discussed – streaming motion past the cylinder with no circulation and irrotational motion around it. In both examples the streamline patterns are completely symmetrical so that in an ideal fluid no net force can arise (D'Alembert's Paradox),

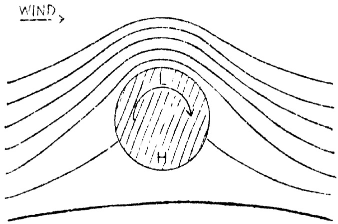

Fig. 12. The Magnus Effect.

but if the two types of motion are combined, as in the case of a wind blowing across a rotating cylinder, symmetry is destroyed and a net force comes into being. This is illustrated in Fig. 12.

In the combined motion the velocity at any point is the resultant of the velocities for the two motions separately. At the position marked L the streaming motion and the rotary motion are in the same direction, whereas at H, diametrically opposite to L, the rotary motion and the streaming motion are in opposition. At L the combined streams are moving rapidly, whereas at H the head-on clash of the two motions produces a region of low velocity, so that, by Bernoulli's theorem, L is a region of low pressure and H one of high

pressure. This means that a force, equal to the difference in pressure between these two regions, acts in the direction HL, so that the motion of a rotating body through the air, or the action of an air stream on a rotating body, produces a cross-current force or lift. This is the Magnus effect, and it is worthy of note that, unlike drag, the explanation does not depend on viscosity. In the case of a tennis ball which is driven forward and also set spinning by an oblique blow of the racket, viscosity enters by making the air in the immediate neighbourhood of the ball stick to the surface and rotate with it, so that 'top spin' (that is, spin in the direction of motion), together with the forward motion, produces a force acting downwards which causes the ball to keep low and dip rapidly towards the end of its flight.

The Rotor-ship. An attempt has been made to utilize the Magnus effect to replace the screw of a ship. In the Flettner 'rotor-ship' the funnel is made to rotate at fairly high speed, thus giving with the natural wind a powerful transverse force which drives the ship along. In the conventional sailing-ship a very large area of canvas is used, necessitating tall masts, complicated rigging and a large, well-trained crew ready to go aloft at a moment's notice. The rotor principle utilizes the force of the wind much more conveniently, but the idea of a rotor-ship does not seem to have appealed to shipbuilders in general and it is now regarded as a scientific curiosity and no more.

The statement is frequently made that the Magnus effect is responsible for the drift of projectiles, that is, the well-known feature in gunnery that a shell swerves slightly even in still air. If the shell is given right-handed spin it always drifts to the right, whereas the Magnus effect alone would make it go to the left. The exact motion of a spinning shell is the most difficult problem in ballistics and cannot be dealt with here, but it can be said that the observed drift is mainly

due to gyroscopic forces which completely outweigh the true Magnus effect.

AEROFOILS

Up to this stage we have not been concerned with the practical problem of flight but have been content to illustrate fundamental principles by describing the motion of the air around simple shapes such as spheres or cylinders. We now turn to the problem which lies at the heart of practical aeronautics – how to use the resistance of the air to overcome weight.

Mechanical flight starts with the fact that a flat plate inclined at a small angle to the horizontal produces more lift than drag when it is moved through the air. This is the sort of experimental fact which everyone discovers for himself at some time or other and which can be developed to achieve quite astonishing athletic feats, as in the art of discus throwing. (In this example the athlete utilizes two principles: that of the top, or gyroscope, of keeping its axis pointing in the same direction, and that of the aerofoil to reduce the net downward pull on the discus and thereby to get greater range.) The average person, if asked to throw a flat object, such as a playing card, as far as possible would quite certainly 'flick' it through the air at a slight angle – some deep-seated mechanical instinct within us makes this choice inevitable – but the next step in the development of the aerofoil is far less instinctive and would certainly not suggest itself to the average human being unless he happened to be a keen observer of birds. This is the advantage gained by giving the plate a slight curvature or camber, so that in shape it resembles the wing of a large bird. Clearly, the factor which matters most in wing development is the ratio of lift to drag, which must be as large as possible, and a slight bend in the plate can easily double this ratio. The highest lift/drag ratio is that obtained by the true aerofoil shape, with

a rounded nose, smooth curved top and a sharp tail. Fig. 13 shows diagrammatically these various shapes and their respective lift and drag forces.

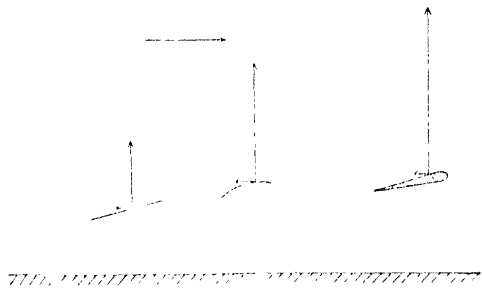

Fig. 13. Lift and drag of an inclined plane, a cambered plane and an aerofoil.

Aerofoil nomenclature. Since the early days of flying, aerofoil shape has been studied intensively in fluid motion laboratories all over the world, and a standard nomenclature has

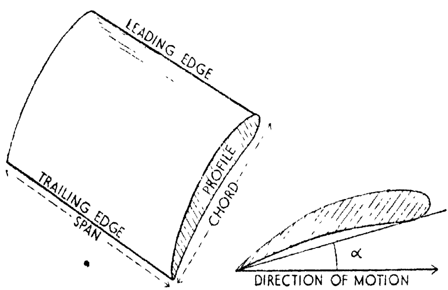

Fig. 14. Aerofoil terms illustrated.

emerged. There are a few technical terms which are essential, and since these are easily defined they are easily remembered. Fig. 14 illustrates these terms. The two principal dimensions

are the *span*, or the distance between wing tips, and the *chord*, which can be defined with sufficient precision for our purpose as the mean length from the leading edge to the rear edge. The product of the span and the chord is the *plan area* S. The ratio of the span to the chord is called the *aspect ratio;* a high aspect ratio means a wing which extends out from the sides of the aircraft much more than along the body, or what one would commonly call a 'long, narrow wing'. The motion of the air will always be assumed to be perpendicular to the leading edge, and the angle at which the aerofoil is inclined to the direction of motion is called the *incidence* or *angle of attack*, and is invariably denoted by the Greek letter α (alpha).

The Lift Coefficient. Lift is a part of the aerodynamic force on the aerofoil, so that we may use Rayleigh's theorem (Chapter III, p. 69) and define the *lift coefficient* C_L exactly as we defined the drag coefficient C_D. Thus

$$\text{lift} = \tfrac{1}{2}\rho u^2 S C_L$$

where u is the speed of the air over the aerofoil (or equally, the speed of the aerofoil through the air), ρ is the air density and S is the plan area. Exactly as in the case of drag, the lift coefficient completely specifies the lift, and it is customary to describe the performance of an aerofoil in terms of C_L and C_D, and rarely, if ever, in terms of the actual lift and drag. In most work with practical aerofoils at speeds well below that of sound (that is, for speeds not exceeding about 400 miles per hour) C_L and C_D can be treated as independent of the velocity. The matter of greatest interest is the variation of C_L and C_D with the angle of attack α.

Fig. 15 does not refer to any aerofoil in particular but is typical of most practical wings. It shows C_L and C_D plotted against α. The lift coefficient is zero at a negative angle of attack and rises in an almost perfect straight line until a certain angle is reached when the curve flattens and ultimately falls, generally rather steeply. That is, the lift of an aerofoil

increases very nearly in direct proportion to the incidence over a certain range of values of α. The straight line portion of the C_L curve is called the *working range* of the aerofoil – it varies from aerofoil to aerofoil, a typical range being from $\alpha = -3^\circ$ to $\alpha = +12^\circ$. The upper value is called the *critical incidence;* when α exceeds this value the lift starts to decrease and the wing is said to be *stalled.* The drag coefficient of a practical aerofoil is naturally very much less than the lift coefficient over the working range and its variation with incidence is quite different, for the C_D values do not fall on a straight line. At the critical incidence C_D is increasing rapidly. The lift/drag ratio usually reaches its maximum, for a practical wing, at a small positive angle of attack. After the critical incidence has been passed, the lift/drag ratio falls very rapidly.

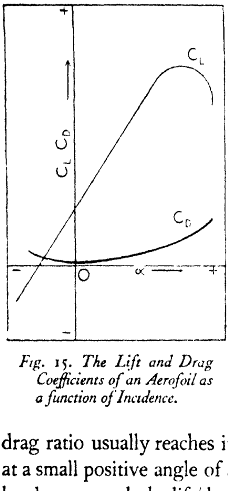

Fig. 15. The Lift and Drag Coefficients of an Aerofoil as a function of Incidence.

The qualities for which the aircraft designer looks in choosing the shape of the wings naturally vary considerably with the purpose for which the machine is being built. Usually, an attempt is made to get a wing with a high maximum lift coefficient, for this furnishes an index of the load which the machine can carry for a given minimum speed, and in particular, determines the safe landing speed of the aircraft. Another way of looking at it is to say that the maximum lift/drag ratio must be kept high, for in normal level flight the lift of the wings supports nearly all of the weight, and the most efficient machine is that which needs the minimum thrust (that is, engine power) to supply this lift and overcome drag. There

is no 'best' aerofoil shape, and the skill of the designer is shown by the way in which he chooses the characteristics which best suit his purpose, sacrificing something here to gain a little elsewhere, and so on.

Stalling. All aerofoils have one feature in common, the decline of the lift/drag ratio once the critical incidence is surpassed. This means that the wing is no longer able to carry out efficiently its primary purpose of sustentation and the aircraft starts to fall, this being frequently followed by a tail spin which may end in a crash if the stall occurs near the ground.

To explain why a wing stalls* we must here anticipate a little the general explanation of lift which will be given later. As in the Magnus effect, the lift of an aerofoil is due to a difference in pressure between the upper and lower sides of the body, and, obviously, to overcome weight the higher pressure must be below. This difference in pressure can only be maintained if the flow around the wing is of a certain type, and anything which tends to upset the flow will, in general, have an adverse effect on the lift. At small angles of attack the air finds little difficulty in accommodating itself to this type of flow, in which the points of separation are near the trailing edge on both the upper and the lower surfaces, giving a very small wake and therefore a low form drag. As the incidence is increased, the flow finds it increasingly difficult to maintain contact, especially on the upper surface, and the point of separation starts to move towards the leading edge. Ultimately a state is reached when the flow, although making contact with the surface at the leading edge, cannot follow the contour of the wing around the steep shoulder and separation occurs at a very early stage, resulting in a broad eddying wake (Fig. 16). This means a considerable increase in drag

* The reader should not confuse this type of stall with that known as 'shock stall', which occurs only at very high speeds. See Chapter VI.

and (for reasons which will be better appreciated later) a large reduction in lift. In full-scale flight the phenomenon is not quite as simple as the above description implies, because the boundary layer becomes turbulent and this, for reasons explained in Chapter III, delays separation, but sooner or

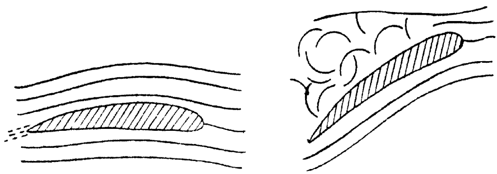

Fig. 16. Flow around an aerofoil, below and above stalling incidence.

later the stall must occur, and in actual flight is often more abrupt than is indicated by the account given here.

Separation of the flow from the upper surface of an aerofoil is sometimes referred to as *burbling*, and the angle of attack at which the phenomenon of the sharp drop in the lift/drag ratio first appears, the *burble point*. Even aerodynamics has its moments of unconscious humour.*

The Slotted Wing. It is not difficult to see that a stall can be exceptionally dangerous if it takes place when the aircraft is near the ground as, for example, in the take-off or when landing, when the wings are producing relatively little lift because the aircraft is moving slowly. An upward gust, or an inadvertent movement of a control, may then tilt the wing so that the critical incidence is exceeded, and because of the low height the pilot has little chance of recovery before the machine strikes the ground.

It is evident that if some means could be found to persuade the flow to cling with greater tenacity to the upper surface of the aerofoil, the stall would be delayed. There have been

* *v.* Lewis Carroll.

numerous ingenious suggestions how this could be done. One method which works well in practice is that of the *Handley-Page slot*, or narrow guide-aerofoil fitted just ahead of the nose. In normal level flight the slot can be closed by the pilot, and the auxiliary aerofoil then forms part of the leading edge of the wing. The separation of the flow which causes stall is essentially due to the air particles in the boundary layer becoming tired and losing their forward momentum because of the fight against the pressure gradient and the viscous drag. The slot directs a stream of fresh vigorously moving air on to the top of the wing to reinforce the boundary layer on its way to the trailing edge. By this means it is possible to prolong the working section of the lift curve by as much as 50 per cent before the stall occurs. Other and less practical means which have been suggested are sucking the recalcitrant air particles back on the wing, or blowing high pressure air from the interior of the wing along the upper surface, or even placing a rapidly rotating cylinder along the leading edge. All these methods are known as *boundary layer control.*

If the aileron at the trailing edge is arranged so that on being depressed it exposes a slot through the body of the wing, the result again is to delay the stall considerably. This device is called a *cut slot*. The extreme use of the slot principle was probably reached when an aerofoil with no less than seven slots was tested in a wind tunnel. It gave an extraordinarily high maximum lift coefficient (that is, it could be tilted to a very high incidence before the stall appeared), but such an arrangement is more of theoretical than of practical interest.

THE ANALYSIS OF LIFT

Anyone acquainted with the rudiments of fluid motion who starts to ponder on the lifting action of a wing will soon con-

clude that it can arise only from a difference in pressure between the upper and lower surfaces of the aerofoil, with the lower pressure necessarily above, so that a wing is both 'pushed' and 'sucked' upwards. This much seems to have been known to Cayley and was certainly explicitly stated by Horatio Phillips in his Patent Specification of 1884, which refers to 'a vacuum (or partial vacuum)' being formed on the upper surface of a convex wing. The first accurate and detailed

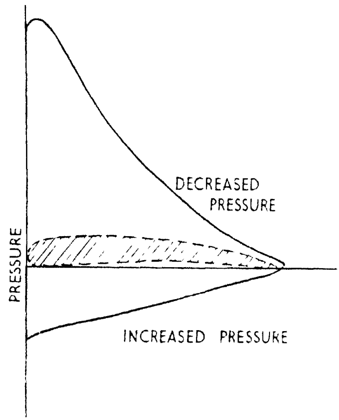

Fig. 17. Distribution of pressure around an aerofoil at moderate incidence.

measurements of the pressure field around an aerofoil were made in England in 1911; these were afterwards repeated in many other countries. The general trend of these measurements is shown in Fig. 17, from which it will be seen that Wenham's statement of 1866 that the greater part of the lift comes from the forward part of the aerofoil is undoubtedly true and that usually (and especially at the steeper angles of attack) most of the lifting force can be ascribed to reduced pressure over the upper surface. Bernoulli's theorem then

indicates that the average velocity of the air near the upper surface must be substantially greater than that near the lower surface, and the question immediately arises how such a velocity field can result from the translational motion of an aerofoil through the air. This is the main problem of aerofoil theory, the solution of which was found by Lanchester.

Lanchester's Circulation Theory. A difference in pressure transverse to the direction of motion exists, as we have seen, when a cylinder or a sphere rotates in a wind and so introduces circulation into a uniform stream. This is the Magnus effect, and in 1877 Rayleigh explained the swerving flight of a 'cut' tennis ball on these lines. In the case of the inclined plane, the arched wing or the aerofoil, there is no source of mechanical rotation, so that it is by no means obvious at first sight that the two phenomena have anything in common.

Lanchester, with extraordinary insight, took the bold step of assuming that the two cases are fundamentally the same, that is, essentially he associated the lift of an aerofoil with circulation. At the time this must have seemed a very dubious assertion, but we now know not only that it is true, but that the statement is valid for bodies other than aerofoils. Even a stone thrown in the air has a circulation associated with it, but unless the body be specially shaped, the circulation is very feeble or, in other words, there is virtually no lift. An aerofoil is a shape which on being propelled through the air generates a strong circulation without causing, at the same time, a large drag, and it is this property, and no other, which makes mechanical flight possible with fixed wings, that is, without using rotating surfaces to create lift. Lift and form drag are, in the end, caused by the same feature, namely, rotation, the difference being that the rotary motion is relatively simple and well ordered in the case of lift and highly complex and disorderly (turbulent) in the case of form drag.

The story of Lanchester's discovery and of its reception by

the scientific world has been told before,* but is worth repeating. In 1894, when a comparatively unknown engineer of twenty-six, he read a paper before the Birmingham Natural History and Philosophical Society on the stability of aircraft, in the course of which he described, in outline at least, his conception of the origin and nature of lift. Three years later, having generally revised this account of his theories, he sought to publish it under the ægis of the Royal Society. To do this meant (and still does) that the paper must be 'communicated' by a Fellow, who is expected to satisfy himself that the work is generally suitable for publication by the Society before he does so. The Fellow to whom Lanchester sent the paper suggested that it should be offered to the Physical Society, and this was done. Before a learned society prints a paper, it seeks the opinion of one or more referees, who are asked to state whether they consider that the work is sound, novel and of sufficient scientific interest to warrant publication in the society's journal. The referees rejected the paper; Lanchester, a disappointed man, withdrew the work and the scientific world at large did not hear of his theories until ten years later when Messrs. Constable and Company published *Aerodynamics: Constituting the First Volume of a Complete Work on Aerial Flight, by F. W. Lanchester*, followed one year later, in 1908, by the second volume entitled *Aerodonetics*. These two books, and particularly the *Aerodynamics*, are now among the classics of aeronautics.

The reader who is unacquainted with Lanchester's original writings may find the reception given to his work inexplicable, but actually there is much to be said in extenuation of the unfortunate referees of the Physical Society who unwittingly cast away a pearl of such price. Lanchester did not originally present his theory in the concise form familiar to-day, and the word 'circulation' is not to be found in either volume of the

* *v. Aerodynamic Theory*, ed. Durand, Vol. I, Division D.

§ 126 AERODYNAMICS.

The whole subject of cyclic motion in the case of a viscid fluid has not been thoroughly investigated. It is evident that to a certain extent the restrictions proper to the inviscid fluid must apply, but since we can generate rotation we are able to induce vortices with a freedom not possible when viscosity is absent.

Basing our argument on the facts as already ascertained, it is evident that if we continuously generate vortices at the right and left hand extremities of the aerofoil, as in Fig. 79, we can regard these vortices as forming in effect, taken in conjunction with the

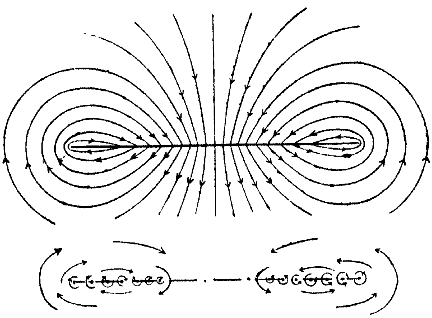

Fig. 83.

aerofoil itself, an obstacle to connectivity, so that, although the vortex dies away after a while, it persists as long as is necessary to permit of a cyclic system being established and maintained.

It is probable that these terminal vortices do not each actually consist of a single vortex but rather of a multiple system of smaller vortices; especially should this be the case with the larger birds, and similarly for mechanical models of any size.

We can conceive that these vortices are formed after the manner indicated in Fig. 83, in which an aerofoil is represented in end elevation with the flow indicated diagrammatically. We

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Page from the first edition (1907) *of Lanchester's Aerodynamics.*

may suppose that the air skirting the upper surface of the aerofoil has a component motion imparted *towards* the axis of flight, and that skirting the under surface in the opposite direction, so that when the aerofoil has passed there exists a Helmholtz surface of gyration. This surface of gyration will, owing to viscosity, break up into a number of vortex filaments or vortices after the manner shown.

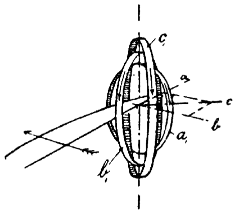


FIG 84

§ 127. Peripteral Motion in a Real Fluid (continued).—The cyclic flow of the vortices to the right and left hand of the aerofoil finds itself superposed on the main cyclic system of the aerofoil, so that the axes of these vortices will not be parallel to the axis of flight as might be supposed, but will take up a resultant direction and may be conceived to spread out as shown in Fig. 85. The compounding of two cyclic systems into a resultant system is illustrated diagrammatically in Fig. 84, in which the circle *a a a* represents the main cyclic system, that whose supporting reaction is concerned in sustaining the load; *b b* represents the cyclic system of one of the vortex filaments, and *c c* the resultant.

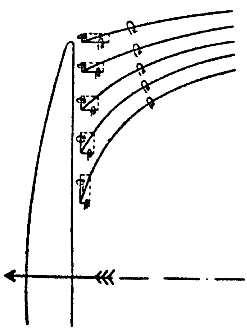
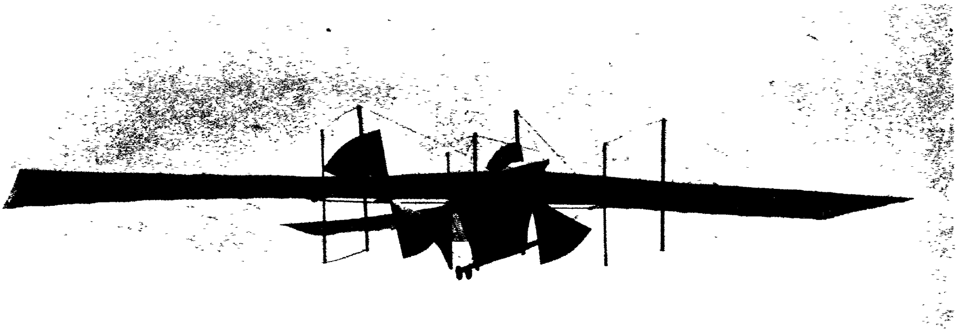
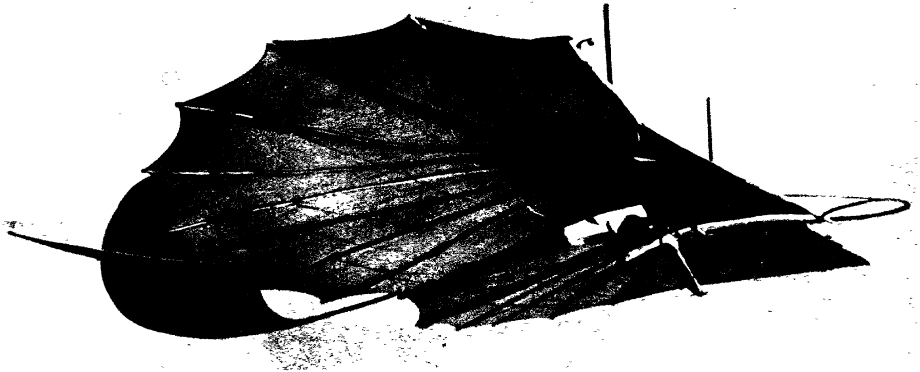


FIG. 85.

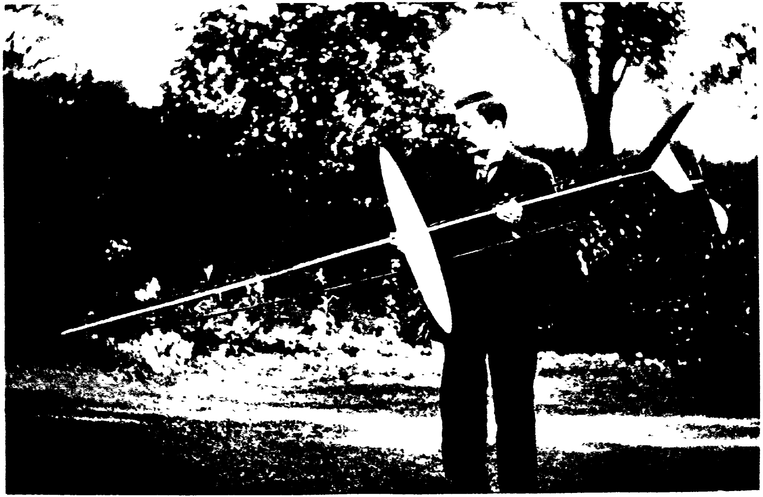
Page from the first edition (1907) of Lanchester's Aerodynamics.



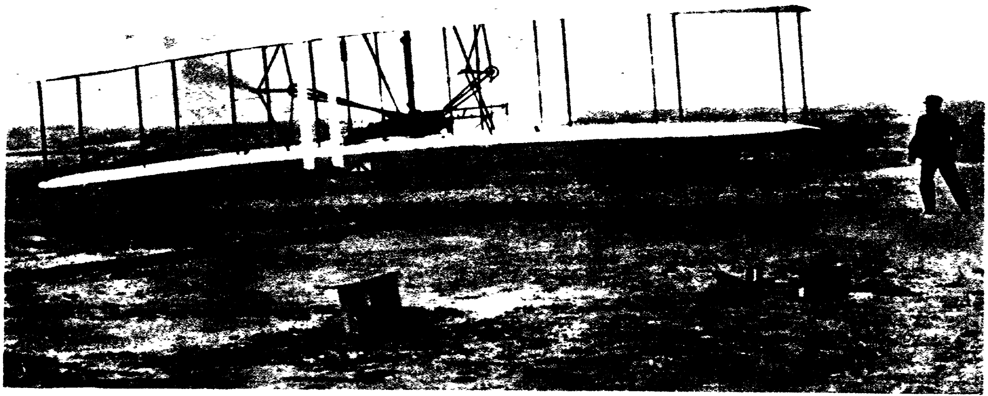
1. Henson's Flying Machine Model built by W. S. Henson and J. Stringfellow in 1844–5 from a design by Henson in 1842 and based on an attempt to imitate soaring birds. The motive power in the actual machine was to be steam



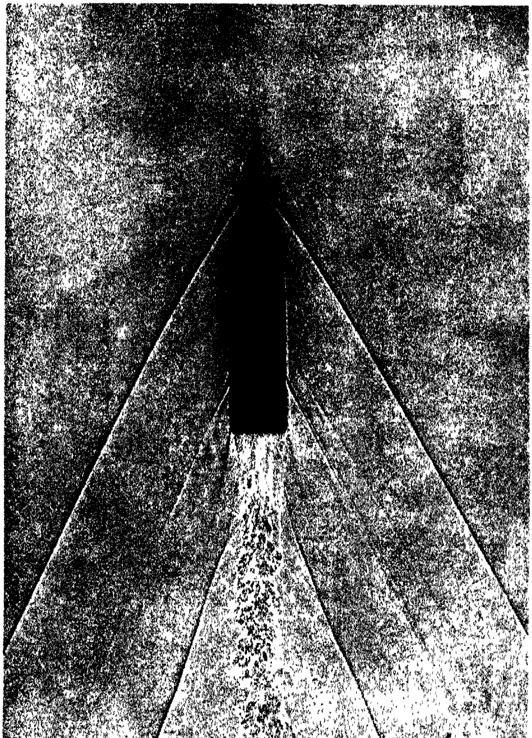
2. Lilienthal Glider, 1895. A machine used by Otto Lilienthal in gliding experiments near Berlin in 1895 and 1896. Flights of considerable duration were made from the summit of a hill. The control was effected by moving the body



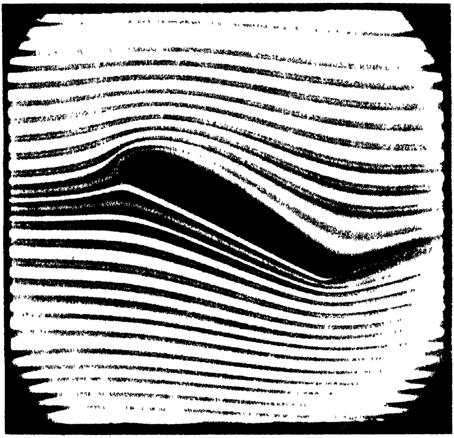
3. F. W. Lanchester with one of his model gliders



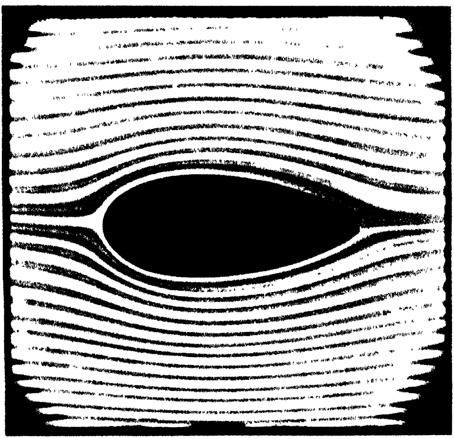
4. The 'First Flight', December 17, 1903. The first controlled and sustained flight in a heavier-than-air machine; Orville Wright piloting, Wilbur Wright on foot. The speed was 10 m.p.h. against a wind of about 22 m.p.h.



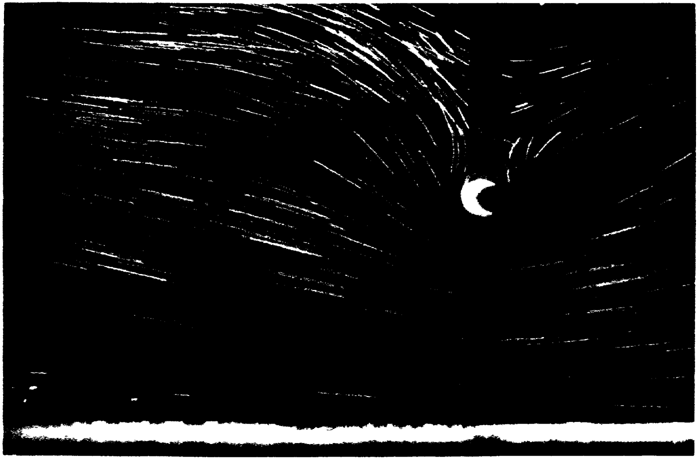
5. Spark photograph of a bullet with a gramophone needle fixed to the nose travelling at supersonic speed, showing the system of shock waves and the turbulent wake



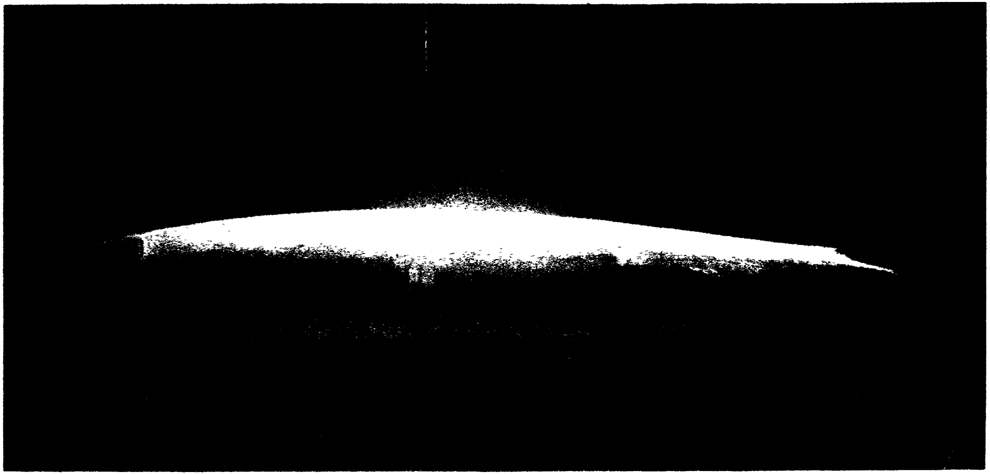
6a. Flow around an aerofoil at low Reynolds number



6b. Flow around a body of 'streamline shape' at low Reynolds number



7. The Magnus Effect. Flow around a rotating circular cylinder in a steady translational flow



8. Model of Airship R80 in a wind tunnel showing laminar break-away near the tail

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Representing diagrammatically the relative strengths of the cyclic systems as the sides of a parallelogram (Fig. 85), we arrive at an indication of the manner in which the vortices will spread as they are left behind by an aerodrome in flight.

Following the matter further we may represent the interaction of the vortices on each other in the manner shown in plan in Fig. 86. This figure is merely a diagram, the motion indicated

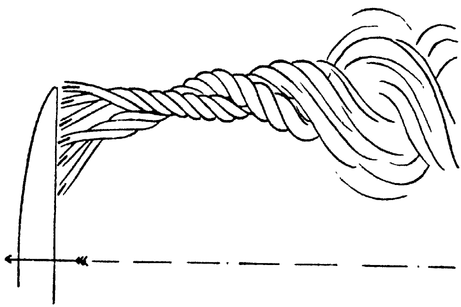

FIG. 86.

being based on the known properties of vortices (§ 93). The filaments will evidently wind round one another like the strands of a rope, being involved in common in the resultant cyclic disturbance. The two vortex trunks springing respectively from the right and left hand wings, owing to their rotation being opposite, do not wind round each other but precess downwards as in Fig. 79. The motion is represented as becoming incoherent in Fig. 86, as undoubtedly must sooner or later be the case.

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Page from the first edition (1907) of Lanchester's Aerodynamics.

first edition of his work on aerial flight. Instead, he used a concept which he entitled a 'forced wave', which is certainly more difficult to grasp than the simple picture of circulation. Even to-day, when we are in the comfortable position of 'knowing the answer', Lanchester's great work is not always easy reading. The analyses of air flow, of lift and resistance are, from the physical aspect, brilliant, suggestive and convincing, but on the mathematical side, hardly ever developed beyond the rudiments. The text often has, for our eyes, a strange aspect because of Lanchester's inordinate love of words coined from Greek roots;* the books are furnished with numerous appendices which are only too evidently jottings from notebooks, covering a variety of subjects from the slide-rule solution of a cubic equation to the theory of gyroscopes, and Lanchester (possibly as a result of his earlier disappointment) never hesitates to challenge an accepted view and to proclaim in vigorous English where he is right and others are wrong. Some light on Lanchester's attitude to the accepted mathematical theories of fluid motion at the time when he conceived his theory may be gained from what he said in later years, at the Wilbur Wright Memorial Lecture to the Royal Aeronautical Society in 1926. 'At the date of this investigation, 1892, I had very little acquaintance with the classical hydrodynamical theory or should have immediately recognized the form of fluid motion in question as a cyclic motion superposed on a motion of translation, which identity was only recognized some three or four years later. ... At the time in question I regarded the system as a "forced wave", that is to say, a wave whose existence or maintenance depended upon a force (or distribution of forces) applied from without. This is a perfectly legitimate view.'

* Such as 'aerodone', 'apteroid', 'peripteroid', 'pterygoid', 'ichthyoid', etc. Only two, 'aerofoil' and 'phugoid' seem to have survived and the latter only in technical treatises on aircraft stability.

This inability, and perhaps unwillingness, to use the methods and results of the mathematical theory was undoubtedly Lanchester's greatest weakness; in the end it robbed him of the full triumph. Yet in many ways it may be accounted a secret of his strength, for unhindered and uninhibited by too deep a knowledge of the classical theory he reached his goal by a more direct, if less certain, path –

Image the whole, then execute the parts
Fancy the fabric
Quite, ere you build, ere steel strike fire from quartz
Ere mortar dab brick!

The original paper of 1897 was never published, but one may imagine that the referees, faced with the difficult task of following a leaping imagination in a subject which had already had more than its fair share of charlatans and pseudo-scientists, were reluctant to recommend publication of a work so unorthodox in its presentation. It is also possible that they did not understand it.

A remarkable feature of Lanchester's work is its isolation. At no time did he form part of a team working at a large institution, nor was his work maintained by funds from external sources to any great extent. Throughout his life he remained an individualist, perhaps the last and possibly the greatest lone worker that aerodynamics will ever see. In later life honours came upon him thickly; he was made a Fellow of the Royal Society and an Honorary Doctor of Laws, he received the Gold Medal of the Royal Aeronautical Society in 1926, the Daniel Guggenheim Gold Medal in 1931, the Ewing Medal in 1941 and the James Watt International Medal in 1945. He died in 1946, at the age of 77; in the words of the James Watt Medal citation, 'a great personality, a brilliant scientist, and a most ingenious engineer'.

The *Aerodynamics* had, however, a considerable measure of success; new editions were published in 1909, 1911 and

1918 and it was translated into German in 1909 and into French in 1914. It opened the way for the final success of the aerofoil theory by Prandtl and his fellow-workers at Göttingen. Prandtl himself paid tribute to Lanchester's work on more than one occasion, and in 1927, when delivering the Wilbur Wright Memorial Lecture to the Royal Aeronautical Society, he summed it all up in the following words: 'Lanchester's treatment is difficult to follow, since it makes a very great demand on the reader's intuitive perceptions, and only because we had been working on similar lines, were we able to grasp Lanchester's meaning at once'.

Let us now look at the modern theory of the aerofoil in its completed form.

The Lanchester-Prandtl Theory of Aerofoils. The starting point of aerofoil theory is the so-called *Kutta-Joukowski theorem*, discovered by the German mathematician W. M. Kutta in 1902 and independently by the Russian scientist N. E. Joukowski in 1905. The result can be enunciated quite simply and is given here more or less in Joukowski's own words: *If an irrotational two-dimensional air stream, of density* ρ, *whose undisturbed velocity is* u, *surrounds a closed curve on which there is a circulation of strength* Γ, *a force of magnitude* $\rho u \Gamma$ *is set up perpendicular to the direction of motion.*

The first and most striking feature of the result is that it contains no reference to the origin of the circulation or to the presence of any solid body, such as an aerofoil, in the fluid. The circulation may be that set up by the cylinder of a rotor-ship, a moving projectile, or the wings of an aircraft, and the result is equally valid. It should be noted, however, that the enunciation refers to two-dimensional motion and, therefore, in strictness, applies only to bodies such as very long cylinders or aircraft wings of very great span, in which case the force is reckoned per unit length of span.

The Kutta-Joukowski theorem is a result in the theory of

ideal fluids; it makes no mention of viscosity and refers only to a force perpendicular to the direction of motion. It has, therefore, no connexion with drag, or the force in the direction of motion, which in any case must be zero in an inviscid fluid. As we shall see, this theorem is one of the great triumphs of the ideal fluid theory in aerodynamics; in this case the omission of viscosity is not completely fatal to the result as it is for drag.

Joukowski Aerofoils. If the circulation around a body is known, the Kutta-Joukowski theorem enables the lift to be calculated and compared with observation, an essential first step towards a rational process of aircraft design. To show a mathematician an aerofoil shape and to ask him to determine the circulation around it is to present him with an exceedingly difficult problem, but the question may be approached in another way.

The reader will have noticed that flow around a long circular cylinder frequently forms a most convenient starting point for discussions in aerodynamics. This is because of the symmetry of the cylinder and the simplicity of the resulting flow, and it is clear that if some way could be found whereby the flow around a body of more difficult shape (such as an aerofoil) could be deduced from that around a cylinder, a great advance would be made. A method of doing this was discovered by Joukowski; it also affords a means of producing aerofoil shapes by a purely mathematical, almost mechanical, process.

The method used by Joukowski is based on what mathematicians call 'conformal transformation' and is essentially the same problem as that of making a flat map of the earth's surface. The reader no doubt is familiar with the representation of the earth known as Mercator's Projection; this is simply a map of the curved surface of the earth on a single flat sheet of paper, in which the meridians and the parallels of latitude on

the sphere correspond to straight lines parallel to the axes of co-ordinates on the flat map. Like all conformal maps, it has the advantage of leaving angles unaltered, but lengths become distorted; it is impossible to produce a continuous plane representation of a spherical surface which is exactly true everywhere. In Mercator's Projection the distortion is most severe near the poles and least in the centre of the map. This is a very useful property for a map of the globe because, generally speaking, we are more interested in the inhabited regions nearer the centre than we are in the barren snow-covered lands near the poles. To the mathematician the problem is one which is studied by the use of imaginary quantities,* or more precisely, by what is known as the 'complex variable'.

In the aerodynamic problem we have to do something very different from the map maker. We want the circle which represents the cross-section of the original cylinder to occupy the centre of the original picture, but in the centre of the map we want not a circle, but a shape which bears a close resemblance to an aerofoil. That is, the distortion must be severe in the centre of the map, but at the edges we want to retain the same undisturbed flow in both the original and the map, so that the distortion must become negligible at points remote from the centre. Joukowski discovered a simple mathematical formula which has exactly these properties; in other words, he found how to transform a circle into an aerofoil shape, with the streamlines of the flow around the circle transforming

* 'Imaginary' quantities are those which involve $\sqrt{-1}$, an expression which has no meaning in ordinary algebra in which the square of any number, positive or negative, is always positive. For problems such as the above, a more general form of algebra is needed, in which quantities such as $\sqrt{-1}$ occur quite naturally. (The term 'imaginary' is an unfortunate relic of the time when mathematicians felt that some mystic quality must be possessed by the square root of a negative number.) Without $\sqrt{-1}$ modern mathematics would lose much of its extraordinary power and a great deal of its interest.

into those around the aerofoil, leaving the flow at points far away from the body unchanged.

Fig. 18 shows examples of some of these transformations. In the first of these the original circle is placed with its centre at the origin of co-ordinates, and the map shows, as would be expected, a symmetrical figure, which is actually an ellipse, so that we now have the flow around a new shape which is more 'streamlined' than a circle. In the second example a certain

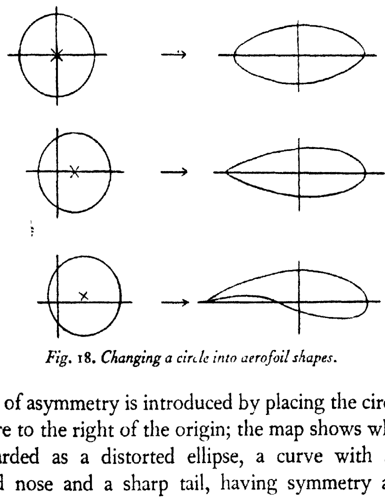

Fig. 18. Changing a circle into aerofoil shapes.

amount of asymmetry is introduced by placing the circle with its centre to the right of the origin; the map shows what may be regarded as a distorted ellipse, a curve with a blunt rounded nose and a sharp tail, having symmetry about a centre line. Such a shape is suitable for a rudder or a strut, and it is evidently a closer approximation to a true streamlined shape than the ellipse. Finally, when symmetry is completely destroyed by displacing the centre of the circle both to the right and upwards, there appears on the map an asymmetrical closed curve which is evidently of the desired aerofoil type since it has a rounded nose leading to a well arched upper

surface and a sharp tail. The flow around a body of this type should therefore approximate to that around a real wing.

Such outlines are called *profiles*, and this process of changing a circle into an aerofoil profile can be done equally well by a draughtsman using a rather long, but not very difficult, equivalent geometrical construction. Joukowski's original method has been greatly developed by other mathematicians, chiefly von Kármán, Trefftz, von Mises and Carafoli, so that it is now possible to obtain almost any aerofoil shape by the method.

To change a circle into an aerofoil shape is purely a geometrical problem, and it is now necessary to consider how the method can yield information about the aerodynamical properties of an aerofoil. The basic idea underlying the use of conformal transformation in aerodynamics is this: the direct problem of finding the flow around a given aerofoil profile is too difficult, but starting with the known flow around a body of very simple shape it is possible to deduce the flow around another body, which looks very much like an aerofoil. In other words, the problem has been *inverted* into that of finding a profile of the desired shape which gives rise to a known flow satisfying the equations of hydrodynamics.

Profiles which have been developed by purely mathematical methods from circles have certain very definite advantages both for theoretical studies and for design work. Thus the lift can be calculated in advance, and the main value of Joukowski's process lies in the fact that even though viscosity is ignored, the results agree closely with reality for moderate angles of attack. Secondly, since the shape itself is given by a mathematical expression, it is possible to introduce any desired variations very simply and to estimate their effect by calculation. As may well be imagined, this method of studying aerofoil shape is popular among mathematicians, to whom it affords an opportunity of exhibiting their virtuosity in

choosing the most significant transformations, and the method of conformal transformation is usually described in considerable detail in theoretical treatises.

The original Joukowski profiles are of no great value for real aircraft – they are much too like tadpoles in shape, all the weight being in the leading portion, with the tails too thin. The development of the method by von Kármán and Trefftz and later by von Mises gives much more practicable shapes, and starting with one profile of the theoretical type, the engineer can now derive any number of related profiles with different camber, thickness, etc., as his problem dictates. The theoretical profiles are the foundations on which real wings are ultimately designed.

Wings of Infinite Span. In the above problems there is one important simplification; the effects at the wing tips are ignored or, as a mathematician would put it, the aerofoil is supposed to have infinite span. This reduces the problem to one of two dimensions only, the thickness and the chord, and this, as always, means a great reduction in the difficulty of mathematical treatment. The results of the theory can be compared with measurements on wings or model aerofoils of great aspect ratio. It has been shown earlier that when the lift coefficient is plotted against the incidence (α), the result is almost indistinguishable from a straight line provided that α is kept small and, in particular, well below the stalling incidence. The mathematical theory of the two-dimensional Joukowski wing in an ideal fluid indicates that, measuring α from the angle of zero lift,

$$C_L = 2\pi \sin \alpha,$$

and since, when α is small and measured in radians, $\sin \alpha$ is practically equal to α, this equation may be written

$$C_L = 2\pi\alpha,$$

which agrees with the practical result in indicating that C_L is

directly proportional to α. The mathematical theory says that the constant of proportionality (that is, the slope of the C_L line on the graph) is 2π, and for a long narrow wing this is very nearly true in practice. This is very different from the case of drag, in which the ideal fluid theory was hopelessly and utterly wrong.

We now turn from these mathematical studies to a consideration of the physical picture of flow over a wing.

Vortices. Vortices have been mentioned before (Chapter III) as regions of concentrated rotation which make up the greater part of the wake behind a bluff body; here it is necessary to interpolate some remarks on their nature and behaviour as part of an orderly motion. Nature provides plenty of examples of well-defined vortices, sometimes on a grand scale, as in the tropical cyclone or revolving storm, in which the wind may reach terrifying force, and sometimes on a minute scale, as in the little whirlpool which forms at the outlet pipe of a bath. A smoke ring is an example of vortex motion in a closed loop, and dimples on the surface of a brook chattering over a stony bed mark the ends of vortices being washed away downstream. In all instances it will be noted that two properties stand out clearly: the concentration of the spinning motion into relatively small regions of space, leaving the main body of the fluid undisturbed, and the stability and relative permanence of such motions.

In the classical theory of vortex motion in an ideal fluid, as developed in the last century by Helmholtz, Kelvin and others, the vortices are idealizations of those found in nature, but have many properties in common with them. The theoretical vortex is permanent and indestructible, always consisting of the same particles and moving with the fluid; it cannot have a free end in the fluid but must either form a closed loop (like a smoke ring) or else terminate on a solid body or a free surface (as in the example of the bath vortex

or the dimples on the surface of a stream). The strength or intensity of a vortex is measured by the circulation around it and is invariable over the whole of its length. In the theoretical treatment we speak of *vortex lines*, *filaments*, *tubes* and *sheets;* a vortex line is supposed to be an axis about which a fluid element is rotating at any instant and a vortex filament (or more simply, a vortex) is a small portion of the fluid bounded by vortex lines, while a vortex tube is the boundary of a vortex filament.

The permanence of theoretical vortices led Kelvin to the idea that atoms, then thought to be indestructible, are vortex rings in the hypothetical æther, since their generation or annihilation would presumably demand a special act of creative or destructive power from without. In a real fluid vortices are formed as a result of the viscosity of the fluid and are eventually dissipated by viscosity.*

To return to the aerofoil problem, since at this stage we are concerned only with the effects of the circulation and not with the problem of its generation, the actual wing may be ignored and replaced by any other arrangement which will give the same circulation. In particular, the two-dimensional wing may be replaced by a long imaginary vortex as a convenient way of providing a circulation. Such a substitution of a column of rotating air for a solid body might have been made before, in the discussion of flow past a rotating cylinder, without making any essential difference to the arguments, and is an example of a device frequently used by mathematicians. The hypothetical vortex which replaces the actual wing obviously must have the same strength as the circulation around the real wing and must always occupy the same place in the fluid as the wing. Such an imaginary vortex is called a *bound vortex;* it cannot consist of the same air particles throughout the motion and is not washed away by the air flow past the wing.

* *v.* Chapter III, p. 64.

The association of real vortices with the flow around a wing was one of Lanchester's most brilliant and inspired contributions to aerodynamics, and will be considered in greater detail in the discussion of flow over a real wing, that is, a wing of finite span.

The Starting Vortex. So far no explanation has been advanced regarding the origin of the circulation. The difficulty here is that, in an ideal fluid, no process can be imagined which will generate circulation from rest, and once a circulation is established, nothing can be done to change it. The origin of circulation, and therefore of lift, lies in the viscosity or internal friction of the air. If air were truly devoid of friction, mechanical flight would be impossible and only buoyant flight, as of balloons, would remain, unless we admitted Lanchester's romantic conception, 'if we had been called into existence surrounded by an atmosphere destitute of viscosity our natural method of locomotion would have been to glide horizontally sustained on the crest of a vortex hoop, a structure which from its immutability would require to be specially created at birth, and would after death continue to pervade the world for all time like a disembodied spirit' (*Aerodynamics*, 1st ed., p. 175).

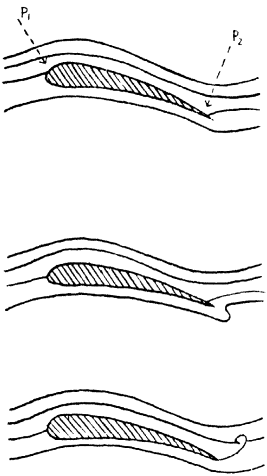

Fig. 19. The start of circulation around a wing.

But we live in a real world, with air mercifully composed

of molecules and therefore possessing internal friction, and we need not speculate about ghostly vortices. Fig. 19 shows what actually happens. When, at take-off, the wing starts moving slowly through the air, the fluid motion at first is almost entirely irrotational, with no circulation. The flow develops two stagnation regions, that is, regions where the fluid is brought to rest relative to the body, one at the nose (P_1) and one (P_2) on the upper surface near the sharp trailing edge. This irrotational motion cannot persist for any length of time, because at the trailing edge the flow is being asked to do something extremely difficult, to get from the underside around the sharp edge to the stagnation point P_2. This means flowing against the pressure gradient, because the pressure rises on approaching a stagnation point (Bernoulli's theorem again!), and in addition, viscosity contributes its own drag. The inevitable result is that the air fails to reach P_2, turns back and coils up into a vortex along the entire trailing edge. This vortex, being across stream, is almost immediately washed off the wing and left behind, but as it moves off, the reaction starts a circulation of the opposite sign around the aerofoil. This is the circulation which causes the lift. Every time the incidence of the wing changes or the aircraft alters speed, a fresh vortex is formed and swept away, and the circulation takes a new value.

The starting vortex is not a mathematical concept but has a real existence. It can be photographed when the motion of the fluid particles is made visible by smoke or other means. It is the seed from which the fully developed circulation grows, and having played its part, is swept away downwind and ultimately disappears far behind the aircraft, as the husk of a seed disintegrates in the soil once the plant is grown.

Wings of Finite Span. So far the account has been entirely of wings of infinite span, that is, of mathematical abstractions of real wings with very great aspect ratio, introduced to

simplify the problem by getting rid of one dimension. The next step is to consider the problem of the finite wing, that is, a wing which has finite span, chord and thickness. It was here that Lanchester made his most characteristic and decisive contribution to aerodynamics.

There are several difficulties to be surmounted. Granted

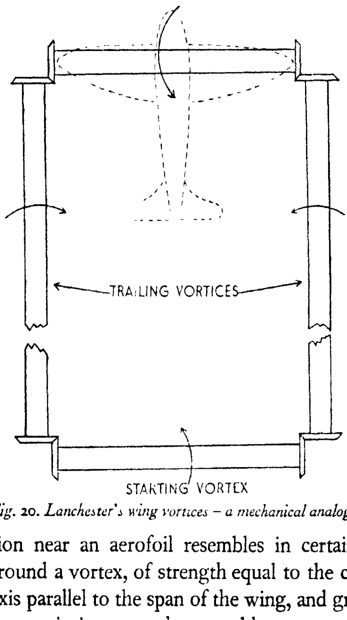

Fig. 20. Lanchester's wing vortices – a mechanical analogy.

that motion near an aerofoil resembles in certain respects motion around a vortex, of strength equal to the circulation, with its axis parallel to the span of the wing, and granted that as the aspect ratio increases the resemblance grows stronger, and more exact, is the conception in any way indicative of what actually takes place? It is no longer possible to imagine the wing replaced by a vortex of finite length, because a vortex cannot begin or terminate in the atmosphere – it must, in this

case, form a closed loop. The first question is therefore: where is the rest of the vortex? The second difficulty is that the strength of a vortex must be the same throughout its length, whereas commonsense says that the lift of a wing, and therefore the strength of the circulation, must drop to zero at the wing tips.

Lanchester attacked these difficult problems with characteristic boldness and asserted that at each wing tip there must be a flow of air from the under surface of the wing to the upper. His actual words are: 'there must be horizontal counter currents formed simultaneously with those in a vertical direction, the horizontal and vertical motions being the horizontal and vertical components of the actual resultant motion of the fluid. We may regard the latter as in the main consisting of two parallel cylindrical vortices, having right- and left-handed rotation respectively, which are being continually dissipated in the wake of the advancing aerofoil'. (*Aerodynamics*, 1st ed. p. 158).

Fig. 20 shows the essential features of Lanchester's concept in the form of a mechanical system. The bound vortex, which indicates the wing, is represented by a solid cylinder rotating with constant speed. The parallel vortices are shown as cylinders geared to the bound vortex; they are free vortices which trail behind the aircraft in flight and, theoretically, they extend all the way back to the starting vortex, thus forming the necessary closed loop. Actually, the starting vortex, being left behind at the start of the flight, soon disappears, its energy being dissipated and absorbed into the atmosphere by the action of viscosity. In practice, therefore, the system approximates more nearly to that represented by the bound vortex and the two trailing vortices. An arrangement of this type is called a ***horseshoe vortex***. Pages 95, 96 and 97, reproduced from the first edition of the *Aerodynamics*, show Lanchester's conception as he gave it to the world.

The trailing vortices actually spring from two *wing-tip vortices* which, in flight, form just inside the wing tips. The existence of such vortices explains, in part, the condensation trails seen extending from the wing-tips of high-flying aircraft in certain meteorological conditions. The core of the vortex is a region of low pressure where the air sucked in expands and suddenly cools, causing condensation of its water vapour content into a mist. At the time when Lanchester evolved his theory such evidence did not exist, and the whole concept is an outstanding example of a man of genius finding the correct solution of a baffling problem without any great weight of experimental evidence to guide him, a feat perhaps more appropriate to the world of ancient Greece than to our own.

The subsequent development of the theory, and in particular the establishment of a sound quantitative basis, is due chiefly to Prandtl. The problem of accounting for the fact that the circulation must decline to zero at the wing tips was solved by the mathematical device of replacing the single bound vortex by a bundle of vortices of different lengths, each with its pair of trailing vortices. Thus ultimately a *sheet* of such vortices streams from the wing behind the aircraft. It can be shown that, at a certain distance downstream, this sheet rolls up at the edges into two vortices, each of strength equal to the circulation around the central section of the wing. Thus although the actual vortex system behind an aerofoil in flight is naturally considerably more complicated than that depicted in Fig. 20, fundamentally the picture is true.

Prandtl's method of allowing for variable circulation along the aerofoil enables any desired distribution of lift to be obtained without disturbing the initial conception and without violating the accepted principles of vortex motion. The best known result obtained in this way is that of the *elliptic wing*, so-called because the curve representing the amount of

lift is a semi-ellipse symmetrically disposed about the centre chord of the wing. Although such a wing is not necessarily elliptic in shape, it is convenient to make it in this way in order to achieve the elliptic lift distribution, the reason for which will be given later. Wings of elliptic or near elliptic plan-form are common features of practical aircraft.

The Physical Picture of the Sustaining Action of the Wing. At this juncture the reader may feel that the above picture of the nature and origin of lift, however it may satisfy the mathematician, is too abstract to be really convincing. Circulation is primarily a mathematical concept, and it was again Lanchester who first pointed out what actually happens to the air as a whole. Lift depends essentially on the power of the wing to deflect air particles downwards, and the reason why a rapid forward motion is necessary in order to maintain a powerful sustaining force is that the wing must continually capture new air particles and drive them downwards. The more rapidly and efficiently it can do this, the stronger is the upward push or recoil. A second difficulty often felt by the newcomer to the subject is that of understanding how the air really supports a weight. The answer is that it does not; in the end the weight of the aircraft is carried by the earth. Lift causes an increased pressure of the air directly underneath the aircraft, but at the surface this pressure, which falls off inversely as the square of the height, is exceedingly small except when the aircraft is within a few feet of the ground. The air thus merely transmits the weight of the aircraft to the ground, spreading it over a large area as it does so.

INDUCED DRAG AND ITS SIGNIFICANCE

The picture drawn by Lanchester of air particles being accelerated downwards by the wing gives rise to an exceedingly important concept in aerofoil theory (also due to

Lanchester), that of *induced drag*, or the part of the total resistance which depends entirely on lift. Induced drag is often regarded by students as a particularly difficult and involved concept, but in reality it is extremely simple once the fundamental facts about flow around an aerofoil have been grasped.

When the air encounters a wing of infinite span it is first deflected upwards and then downwards as the aerofoil passes, and the diagram of vertical velocity is completely symmetrical. The same thing happens at the centre of a wing of finite span, but at the wing tips there are now the trailing vortices to be

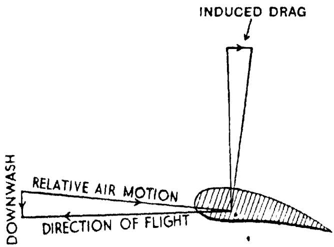

Fig. 21. Induced Drag.

taken into account and these cause, or induce, an additional downward motion of the air. Suppose now, to simplify ideas, that the aircraft as a whole is flying horizontally. At the wing itself the induced downward motion makes it appear that the air is approaching along a slight downward slope, or equally, that the wing is moving at a slightly reduced incidence, just as if the air had 'given way' a little under the pressure of the aerofoil. This means that the lift force, which is always perpendicular to the direction of motion, appears to be bent back slightly and is no longer vertical, that is, perpendicular to the direction of flight (Fig. 21). Such a backward deflection of the lift force is exactly what would be experienced

if an additional drag, or force in the direction of motion, had appeared. This is the so-called induced drag, a name introduced by the German mathematician M. Munk, and now universally accepted.

The discovery of induced drag and the realization of its importance is due to Lanchester, by whom the whole matter was stated quite clearly and for the first time in the *Aerodynamics.* Induced drag is the toll levied by Nature for the privilege of flying. What Lanchester showed was that the weight of an aircraft cannot be carried without the formation of the trailing vortices, whose maintenance naturally means an expenditure of power, which must ultimately come from the engine. Part of the power generated by the engine is thrown away because of the inefficiency of the propeller and part goes to overcome the form drag and skin friction of the aircraft as a whole, but there remains the important contribution to be made to counteract induced drag, which represents the effort required to keep the aircraft from falling to earth under its own weight. The function of lift is to overcome weight, and so induced drag may be correctly described as that part of the total resistance which depends entirely on the lift. No other form of locomotion suffers from this disadvantage; the weight of a car or locomotive is balanced directly by the reaction of the earth, and a ship moves on the surface of the sea because of its buoyancy, but an aircraft, heavier than air and a long way from the solid earth, must pay a price for its power to move freely in the skies. Fortunately, the horsepower required for this is not excessive, especially at the higher velocities, and induced drag places no final limit on speed.

In technical work, induced drag is so important that it must be separated from the rest of the resistance, and it is customary to speak of the *profile drag* of a wing as that part of the resistance which arises because an aerofoil is a solid

body, imperfectly streamlined and subject to the action of viscosity, moving through a fluid. Thus:

$$\text{Total resistance} = \text{profile drag} + \text{induced drag},$$

or, otherwise:

$$\begin{aligned}\text{profile drag} &= \text{form drag} + \text{skin friction}\\ &= \text{total resistance} - \text{induced drag}.\end{aligned}$$

There is a very distinct difference in the way in which these two forms of drag vary with speed. For a given shape of wing, profile drag in level flight increases approximately as the square of the velocity, but induced drag in the same conditions *decreases* as the square of the velocity. An approximate formula for the induced drag is

$$\text{induced drag} = \frac{1}{3} \cdot \frac{1}{\frac{1}{2}\rho u^2} \cdot \left(\frac{\text{lift}}{\text{span}}\right)^2,$$

where $\frac{1}{2}\rho u^2$, as usual, is the dynamic pressure exerted by the air on the wing.

This expression shows the advantage of a large aspect ratio when the load carried is heavy – in other words, the larger the span, the better the wings are able to get a grip on the air. The ratio lift/span is known as the *span loading*. In straight level flight the lift of the wings is very nearly the same thing as the total weight of the aircraft, but it would not be enough to calculate the required horse-power of the engine simply by substituting weight for lift in the above expression. To maintain height during a turn with the machine 'banking', for example, means that additional lift is needed to provide for a vertical component of lift equal to the weight; this implies an increase in the induced drag and therefore in engine power. The designer must take all such factors into consideration and supply an adequate margin of safety.

One of the most important results of the induced drag theory was obtained by Lanchester and given in the *Aerodynamics*. It is that *the drag of an aircraft is a minimum when the induced drag is exactly half the total drag.* For a given aircraft, there is a definite speed at which the total drag is a minimum, but this is usually rather low for a modern aeroplane, which can afford to call on additional engine power to cruise at a higher speed.

With a conventional fast monoplane of clean lines and highly finished surface, induced drag becomes small when flying at top speed, probably less than one-tenth of the whole resistance. A substantial fraction of the total drag, perhaps one-third, comes from the form drag of the wings, tail planes, fuselage and particularly the engine housing, but the greater part of the resistance developed by such machines comes from skin friction. This is a remarkable tribute to the skill of the designer in achieving a close approximation to a true streamline shape.

The Reduction of Drag. The resistance offered by a wing, as we have seen, is made up of two parts, induced drag and profile drag. There is no such thing as the 'best' wing shape, because different properties are needed for different types of aircraft, but having settled on the total lift required and determined the span, the designer may legitimately inquire what plan-form he should give to the wing in order to achieve the minimum induced drag. This, in effect, amounts to what is called the 'second problem of aerofoil theory', namely to determine the distribution of lift over a wing of given total lift and given span which will give the least induced drag. In 1919 Munk proved that, for a single wing, the minimum induced drag occurs when the induced downward velocity has the same value at all points along the span. The 'elliptic wing' has exactly this property and in this sense is the 'ideal' wing, but the condition is by no means critical and small

departures from the exact elliptic lift distribution make very little difference to the final result.

The problem of reducing profile drag has been extensively investigated in aerodynamical laboratories since early days, and many hundreds of aerofoils have been examined to see if a shape can be found which is superior to others in this respect. Boundary layer control is directed towards the same end, but during the war much attention was given to what are known as *laminar-flow aerofoils*. These are aerofoils in which by careful design the boundary layer remains non-turbulent for much greater distances from the leading edge than with an ordinary aerofoil, that is, the transition point (*v.* Chapter III, p. 65) is pushed back towards the tail, with a considerable reduction in skin friction. An aerofoil with this property (known as a Piercy aerofoil and which is derived by a geometrical process, not from a circle but from a related curve, the hyperbola) was tested in 1939 at the National Physical Laboratory at a high Reynolds number. This gave the amazing result that its drag at zero incidence was actually less than that of a smooth flat plate, owing to the fact that the shape had been so skilfully calculated to eliminate adverse pressure gradients that the flow remained laminar over a very considerable part of the surface despite the high Reynolds number.

Laminar-flow aerofoils are particularly appropriate to modern high-speed jet operated aircraft, since in these machines there is no disturbed flow from the propeller going over the wing, and also because at these very high velocities the need to eliminate every ounce of unnecessary drag is predominant. The testing of such aerofoils must be done in special wind tunnels which are as free as possible from turbulence, and very few of these exist. As yet it is too early to say much about this development which, no doubt, will have considerable influence on the problem of high-speed flight.

PROPELLERS AND WINDMILLS

A *propeller*, as its name implies, is a device for moving fluid, or more precisely, for adding momentum to a fluid. The term *airscrew* is now coming into more general use, but a more logical usage would be to retain the term 'airscrew' for any mechanism with radial blades rotating about an axis and to differentiate between propellers, used to supply energy to the air, and windmills, used to absorb energy from the air.

The engine of a conventional aircraft provides a torque, or force producing rotation of a shaft. The propeller is the device which converts the engine torque into a *thrust*, or a force in a straight line, which must be large enough to overcome the drag of the entire aircraft and also to provide sufficient additional power to enable the aircraft to climb. It does this by making use of the resistance of the air, whose reaction on the blades produces two forces, one of which is the desired thrust along the axis of rotation while the other acts as brake on the engine shaft. In steady flight the torque developed by the engine exactly balances the braking action due to air resistance.

The thrust of the propeller is obtained by giving a backward velocity to the air with which it comes in contact, and to do this effectively the propeller blades are given first an aerofoil shape and then a twist. The propeller works exactly as a wing does, by making use of the lift-producing property of an aerofoil, but with this exception, that when an aerofoil is used as a wing it is required to produce the maximum force at right angles to the direction of motion, but when used as a propeller the requirement is for a maximum force in the direction of motion. Now the motion of a propeller through the air is compounded of a rotation about its axis together with the forward motion of the aircraft, so that the blades themselves move forward on spirals. To yield as large a thrust with as little resistance to motion as possible the aero-

foil-shaped blade must be twisted throughout its length to take account of the spiral path. The angle between the chord of the blade and the plane of rotation is called the *pitch*; it varies over the blade because of the twist, and is least at the tip.

The circle swept out by the blades is called the *propeller disc*, and the column of air which passes through the disc is known as the *slipstream*. Immediately in front of the propeller disc is a region of low pressure, into which air is sucked, and immediately behind the disc there is a region of high pressure, from which the emerging air is blown. On approaching the disc the air velocity increases steadily and continues to rise for some distance behind the disc, so that the maximum velocity in the slipstream is found, not at the propeller itself, but some distance downstream. The slipstream contracts after passing through the disc to a minimum cross-section (*vena contracta*), at which the streamlines all become parallel; this is the position at which the maximum velocity is reached. The actual thrust developed is caused by the difference in pressure in front of and behind the propeller disc.

The above account is naturally a somewhat idealized picture of how a propeller works; a complete statement would require a great deal of space and some rather involved mathematics and is hardly of sufficient general interest to warrant inclusion in a book of this type. Historically, propeller theory has developed along two distinct lines, the so-called *momentum theory* and the *blade element theory*. The momentum theory is essentially the work of the English scientist W. J. M. Rankine, and was first published by the Institute of Naval Architects in 1865. This treatment is mainly concerned with the motion of the fluid in which the propeller is working, and the problem is essentially that of finding the system of forces which when acting on the propeller creates

this motion. The blade element theory was initiated by another Englishman, W. Froude, to whom the science of ship construction owes so much, and was published in 1878, also by the Institute of Naval Architects, but the major development of the theory is undoubtedly the work of S. Drzewiecki from 1892 onwards. This method concentrates attention on the forces experienced by the blades in moving through the air and therefore leads naturally to a close study of the actual blade shape. For some time the two theories appeared to be fatally opposed in certain respects, but with the advent of the Lanchester-Prandtl aerofoil theory and a proper appreciation of the factors which go to make up lift, a reconciliation has been effected. It was Lanchester who first pointed out that, as a consequence of the aerofoil nature of the blades, vortices must spring from the tips of a rotating propeller and travel downstream in spiral paths, a conception which has since been amply confirmed by actual photographs taken of propellers working in water. After Prandtl's completion of Lanchester's largely intuitive theory had become well known in the scientific world in the early 'twenties numerous workers, chiefly Betz, Glauert, Pistolesi and von Kármán, brought the whole matter to a reasonably satisfactory conclusion.

The propeller is a most useful but far from ideal means of aircraft propulsion. For reasons which will be better appreciated later (Chapter VI) there is an economic upper limit to the speed at which the blades can rotate and still work efficiently. This limit is reached when the tip speed approaches that of sound, and it is probable that a forward speed somewhere between 500 and 600 miles per hour represents the limit which a propeller-driven aircraft can attain in still air at altitude. Apart from these considerations, however, the propeller shows serious faults of an aerodynamic nature even at moderate speeds. The vortex system developed by the blades represents so much wasted power, for the rotational

motion adds nothing to the thrust, and the disturbances thus created can interfere seriously with the otherwise smooth flow over the wing.

Some of the faults of the propeller can be mitigated by the use of *variable pitch.* Generally speaking, the most critical moments of a flight are at take-off and landing, and take-off can be particularly difficult with a large heavily-loaded long-distance machine. It is during this period, and when gaining height over the aerodrome, that the pilot needs every ounce of thrust his engines can supply. One of the salient features of an aero engine of the conventional type is that it develops its full power only at top rotational speed, which in turn is largely determined by the resistance offered by the propeller. At take-off the pilot is glad of a propeller with low pitch since this offers a minimum of resistance, but at altitude a high pitch is needed to cope with altered conditions and to maintain a high, economical speed. Variable pitch may easily double the thrust available at take-off, in addition to allowing the pilot to choose the propeller characteristic best suited to the altitude at which he happens to be flying.

Windmills. These picturesque and often stately ornaments of the landscape represent one of man's oldest and most successful attempts at aerodynamic design. There are two types of windmills; those which are occasionally mounted on aircraft as a subsidiary source of power and those which stand on the ground and rely on the natural wind to supply their energy. Of the first type all that need be said is that in this case the wind speed is high and unfailing, and in order not to spoil the characteristics of a good machine the efficiency of such a windmill should be as high as possible. Here the usual propeller theory may be applied.

The windmill of tradition is in quite another category. Here the drag does not matter provided that the whole structure can withstand a gale, but what does matter is the amount of

useful work which the mill can supply in relation to the cost of erection and maintenance. In the Middle Ages even this question hardly arose, because with the exception of water power the windmill had no serious competitor. It is natural to imagine that, with a wealth of aerodynamic knowledge at our disposal, it would be possible now to improve very considerably on the traditional mill.

Windmills mounted on the ground generally rotate with a tip speed between one and two times the natural wind speed. These are what are known as slow-running mills, and the tendency to-day is to make them in the form of a multi-bladed airscrew. A higher speed of rotation can be obtained with the use of a two-bladed airscrew, but this sort of fast-running windmill requires very careful aerodynamic and structural design if it is to compete with the slow-running type. The great advantage of the slow-running type of windmill for operation on the ground is that, owing to its larger blade angles and blade area, it will operate in lighter winds than the fast-running type.

It is difficult to obtain a real measure of the efficiency of a traditional windmill, but opinion seems to be that, for the purpose for which it was intended, the old mill could do its job admirably and that even the knowledge of to-day could not greatly improve it, except, of course, in the matter of bearings and friction generally. Modern aerodynamic theory, in other words, does not really enter until things start to move fast, and for the slow, peaceful rotation of the traditional windmill age-old craftsmanship still suffices.

The Autogyro. One of the most striking and least expected applications of the windmill is the *Autogyro*, in pre-war days always a favourite with the spectators at air displays. *Helicopters*, or machines which derive their lift from one or more propellers with their axes vertical, have long been a dream of mankind. Leonardo da Vinci sketched one such machine and those who

read Jules Verne in their boyhood (and who of that generation did not?) will doubtless remember his remarkable flying machine which, with the aid of a host of whirling airscrews, defied gravity, and also, it is to be feared, most of the laws of mechanics at the same time. There is, of course, much to be said for the helicopter concept; one of the great drawbacks of the aeroplane is that it requires a very large area of carefully prepared countryside to enable it to take-off and land with safety.

At first sight it would not seem to be a difficult matter to design a machine to ascend and descend vertically, or nearly so, and no doubt this is why the project has attracted so many inventors, but there are formidable obstacles to be overcome. One of the main difficulties is that of preventing such a machine from rolling over, owing to the fact that if an attempt is made to get forward motion by tilting the axis of the propeller slightly out of the vertical, an advancing blade will then develop more lift than one which is retreating, creating an out-of-balance turning force. This could be overcome by giving the machine two concentric horizontal propellers rotating in opposite directions, but the neatest, although least obvious, solution is to allow the blades to flap as they rotate. When properly arranged this has the effect of decreasing the angle of attack of the advancing blade and increasing that of the receding blade, so that the lift is equalized.

The autogyro obtains its lift from a windmill with such a flapping motion fitted above the machine. The thrust which drives the aircraft forward is derived from an ordinary propeller at the nose, and in flight the windmill rotates freely in the wind caused by the motion of the machine through the air. In doing so it imparts to the aircraft the necessary lift, and in this way the machine is enabled to ascend and descend quite slowly along a very steep path. The latest version, the so-called 'jumping gyro', leaps vertically (or nearly so) from

the ground; this is done by giving the windmill a very high rotational speed when on the ground by connecting it to the engine and reducing pitch; the clutch is then slipped and the pitch restored to its usual value, giving a large increase in lift which causes the machine to take-off with virtually no initial run.

Thus, by a strange coincidence, one of man's oldest conceptions of mechanical flight has been achieved by adding to the aircraft as we know it another ancient aerodynamic device, that of the windmill. In efficiency the autogyro cannot compare with the modern multi-engined aircraft; it has a low forward speed which apparently cannot be materially increased without endangering its most characteristic feature, that of a low speed of descent, and both its weight-carrying capacity and its size appear to be strictly limited. Recently, developments have been made in the design of the true helicopter, but at this stage it is impossible to say with any certainty how far the dreams of Leonardo and Jules Verne may yet come true.

CHAPTER V

Stability in Flight

THE preceding chapters have indicated how, one by one, the individual problems of aerodynamics have yielded to treatment, but this does not mean that the problem of flight is thereby solved. There remains the extremely important question of the stability of the aircraft as a whole.

The Controls of an Aircraft. An aircraft is a body which moves in three-dimensional space, and if a system of mutually

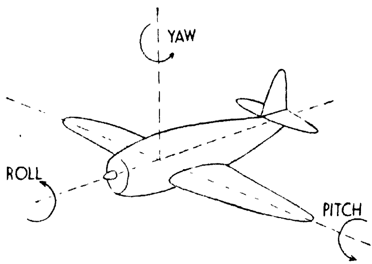

Fig. 22. The motions of an aircraft in flight.

perpendicular axes be imagined in the machine itself, it will be seen that it can *pitch* about the cross-wind axis, *yaw*, or change direction in the horizontal plane about another axis, generally vertical, and *roll* about the third longitudinal axis (Fig. 22). The pilot must be given means of causing all these movements when required.

The instrument panel of a modern aircraft presents a picture of considerable complexity, but the essential controls are few and simple. The most obvious is the *rudder*, which

performs the same function as its familiar counterpart on a ship, causing the aircraft to yaw to right or left. A second rudder, called the *elevator*, varies the load on the tail, making the nose of the aircraft rise or fall and the machine as a whole to ascend or descend – this is the pitch control. The *ailerons* or hinged flaps on the rear edges of the wings move in opposition (one up, one down, or both central); these vary the lift on the wings and thereby cause the machine to roll. The remaining control is the engine throttle, by which the thrust is varied. The controls do not act independently; thus a certain amount of rudder correction is needed when the ailerons are operated and the machine is made to 'bank'.

STATICAL AND DYNAMICAL STABILITY

When in flight, an aircraft is subject to a rapid succession of random disturbances, usually small, although on occasions they can be severe. These disturbances are due to the inhomogeneity of the atmosphere which gives rise to horizontal and upward and downward gusts of wind. If the flight is to continue without mishap and along a pre-selected path, the effect of such disturbances must either be negligible or else corrected as soon as they arise. In theory at least, the pilot is there to apply any corrections, but he is entitled to ask that the machine be designed to assist him in the task. The situation is like that which arises in cycling; the bicycle is subject to a continuous stream of extraneous impulses such as those caused by bumps in the road or by gusts of wind, and although the rider is barely conscious of the fact, these forces are continually being neutralized by an equally rapid succession of small adjustments. Usually these mean no more than slight twitches of the handlebars, causing small changes in the force set up by the reaction of the front wheel on the ground, and the reason why it is impossible to ride a bicycle for any

distance with the handlebars locked is that in this case the rider is unable to supply the corrections necessary to maintain balance.

The task of flying would be impossibly difficult if the pilot himself were obliged to detect and correct each small disturbance as it arose. A human being has a definite time of reaction to any external stimulus, during which he does nothing whatever. (This time varies from individual to individual and a fair average is one-fifth of a second.) Cycling is possible because, first the rider is in intimate contact with the machine and can apply his corrections without any intervening mechanism, and secondly, the motion is so slow that time of reaction does not really matter. In an aircraft the pilot has to deal with a much more complicated set of forces and has to apply his corrections through a mechanical system and, above all, things happen so rapidly that he has little chance of keeping up with the succession of events. All this means that a practical machine must possess some inherent stability, that is, it must be able to look after itself to a certain extent. Failure to appreciate this was probably the main cause of Lilienthal's fatal crash.

The word 'stability' needs to be defined rather carefully here, and it is necessary to distinguish between what is called *statical* and *dynamical stability*. These are best explained by examples. A bicycle in the upright position is statically unstable, but when ridden it has dynamic stability. A tricycle, on the other hand, has both static and dynamic stability. In the first example, if a bicycle were carefully placed upright and left to itself, even the slightest push would cause it to fall – in technical language, there are no forces present of sufficient magnitude to bring it back to its original position. The tricycle, on the other hand, cannot be upset by a small disturbing force. Statical stability simply means that the forces resulting from a *small* displacement tend to restore the

system to the original position, and statical instability is just the reverse, that is, the forces left after the *small* displacement operate so as to increase the displacement from the original equilibrium position. The word 'small' is essential; any system can be upset by a sufficiently large displacement.

Dynamical stability is much more difficult to define in a phrase, and the concept is best explained by examples. Consider a sensitive measuring instrument, such as a volt-

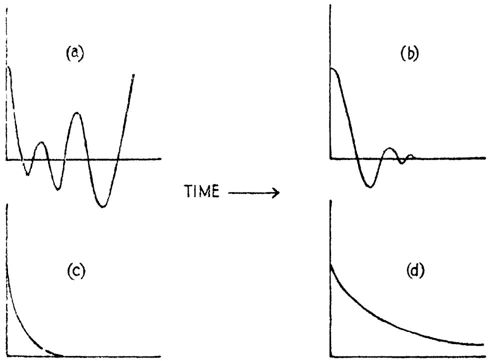

Fig. 23. Illustrations of dynamical stability.

meter or ammeter. This consists of an arrangement in which a pointer moves over a scale in response to an external stimulus, and when the instrument is switched into a circuit, the pointer appears to move rapidly to a mark on the scale and stay there. If the motion of the pointer is considered in detail it will be seen that it must be one of two types: either the pointer slows down as it approaches the final reading and stops exactly at the reading or else it first overshoots the end point, then slightly undershoots and so on, oscillating about the final reading with diminishing amplitude until it finally

stops. In a well-made instrument all this happens so rapidly that the eye cannot follow the motion.

A system of this sort, in which the effect of a sudden disturbance is to make the system take up a new position or return to the original position, either directly or with diminishing oscillations, is described as dynamically stable, and the motion is said to be *damped.* Damping in a mechanical system is usually effected by introducing friction, but other means also exist. Fig. 23 illustrates what can happen to a mechanical system in which the amount of damping is varied. Suppose the system is given a sudden impulse which causes a departure from its equilibrium state. The restoring forces which are brought into play usually result in oscillations and without adequate damping it is possible that these oscillations will not die away but grow larger with time, until eventually something catastrophic occurs. This illustrates an unstable system; a car with damaged or worn stearing gear which develops wheel wobble may behave like this and ultimately get completely out of control. A moderate amount of damping produces the effect shown in (*b*), in which case the system is said to be stable and to execute *damped vibrations*, which ultimately die away completely and the system becomes quiescent again. Fig. (*c*) shows what is known as *critical damping;* the system attains its equilibrium position again without any overshoot and the motion is then said to be *dead-beat.* Finally (*d*) illustrates an overstable or very highly damped system, in which case the return to the equilibrium position is very slow. An instrument with such characteristics would normally be described as 'insensitive' and would be suitable only for problems in which the investigator wishes to exclude transient effects as far as possible.

An aircraft must be statically stable before it can be flown at all, but the amount of dynamical stability which it exhibits can be varied according to the purpose of its use. In a fighter

plane, for example, the pilot must be able to impose his will on the machine very easily and such aircraft have a relatively low degree of dynamic stability. They are 'lively' on the controls and consequently require greater skill in handling than less specialized craft. Large inherent restoring forces, on the other hand, make a machine unsuited for aerobatics but well adapted for more sedate purposes, such as carrying passengers.

THE THEORY OF AIRCRAFT STABILITY

It is a curious fact that the theory of aircraft stability had been firmly established long before aircraft became a common feature of the skies. This is mainly because the theory really has very little to do with fluid motion proper, being essentially a study in 'rigid dynamics', a subject which had reached a high state of development by the end of the nineteenth century. The systematic investigation of dynamical stability in general owes much to the Cambridge mathematician, E. J. Routh, who published a memorable essay on this subject in 1877. This theory was first applied to aircraft stability by G. H. Bryan and W. E. Williams in 1904. In 1908 Lanchester made his own striking and original contribution in the second volume of *Aerial Flight*, the curiously titled *Aerodonetics*, which was followed in 1911 by another well-known book, *Stability in Aviation*, by G. H. Bryan. The later developments were mainly due to Bairstow and Glauert.

The theory of aircraft stability is necessarily complicated and involves some very heavy mathematical work. It is therefore unsuitable for inclusion at any length in a book of this type and all that will be done here is to give a short account of some of the basic results in general terms. This is greatly simplified by the fact that it is possible to deal separately with longitudinal or fore-and-aft stability, and lateral stability.

Longitudinal Stability. A conventional aircraft in steady level flight can be thought of as a see-saw balanced at its centre of gravity, with at one end the lift forces set up mainly by the wings with contributions from the fuselage and propeller, and at the other end the lift forces due to the tail planes. An obvious first condition for sustained level flight is that the moments of these forces about the centre of gravity must balance, the greater lift of the wings being compensated by the much longer lever arm of the tail planes. This, however, is clearly not enough, and a second condition is that if the balance is disturbed, the rate of increase of the righting moment due to the tail planes must be greater than that due to the wings, fuselage, etc.

Thus the tail planes are mainly responsible for maintaining longitudinal stability. In general, the wings, fuselage and propeller tend to upset stability and the function of the tail is to correct this tendency. The fact that an aircraft can be stabilized longitudinally by the tail alone was discovered by Alphonse Pénaud about the year 1870. Pénaud demonstrated his conclusions with small paper-winged model aircraft, some of which were fitted with a propeller driven by twisted rubber, just as in model aircraft of to-day. He succeeded in producing completely stable flight for the not inconsiderable distance of 50 yards, a feat recognized by Lanchester in the *Aerodonetics* with the words 'To M. Pénaud belongs the credit of producing the first stable model of a type that may be regarded as an embryo flying machine', but this is doing something less than justice to Cayley.

So much for static stability; the study of the exact response of an aircraft to a sudden disturbance, such as an upward gust, is a problem of dynamical stability and much more difficult. It is clear from first principles that the machine will tend to swing into the relative wind, but the exact way in which it does this requires careful examination. The tail produces a

damping effect, and by adjustment of the relevant factors in design it is possible to arrange matters so that in normal flight the aircraft responds to a sudden upward gust by a very quick, almost dead-beat, movement into the wind. If the gust catches the machine near the stalling point the effective damping is reduced and a short period oscillation results. The effect of the tail in normal flight is therefore to prevent the oscillations becoming dangerously large as well as that of supplying a force in the correct direction.

*Phugoids.** An oscillation of quite another type was discovered by Lanchester and discussed at considerable length in the *Aerodonetics*, mainly as an example of the exchange of energy between the potential and kinetic forms. The effect of a short-lived upward gust on the longitudinal motion of an aircraft in level flight is to leave the machine with its nose slightly down, causing it (unless corrected) to swoop downwards, gathering speed as it does so. The enhanced speed brings about an increase in lift, which ultimately stops the descent and causes the machine to climb again. At the top of the climb the machine has insufficient lift to maintain itself and if left alone will fall again, and so on. This is the phugoid oscillation, consisting of a wave-like path or a series of loops, and either of these types of motion can be easily obtained with model aircraft. Lanchester himself used models to verify his theoretical analysis; his model 'aerodrome' (Plate 3) although looking strange to our eyes achieved some quite long flights. The importance of the phugoid theory is that, despite the simplified nature of the assumptions, it succeeds in giving a reasonably accurate picture of the possible motions of a body in free flight, especially at low speeds. The accurate quantita-

* This odd-looking word was coined by Lanchester from φυγη and ειδος, literally 'flight-like', but this was a 'howler'. φυγη implies flight only in the sense of being a fugitive and not as of a bird. Despite this, the word has survived in the literature of aerodynamics, possibly because it is short and easy to pronounce.

tive analysis of the motion requires much more detailed consideration.

Lateral stability. If an aircraft experiences a disturbance which causes it to roll, it is evident from first principles that the wings will, in general, act to stop the rolling motion. There are, however, certain points about the action of the wings which require more detailed consideration.

When a roll starts the wing moves vertically (or nearly so) through a horizontal air stream. To the wing the air flow will then appear to be inclined to the horizontal at a small angle, so that the upward-moving wing will be effectively at a reduced incidence and the downward-moving wing at an enhanced angle of attack. This means that the wings will

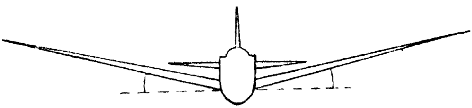

Fig. 24. The Lateral dihedral.

generate different amounts of lift which in combination check the rolling motion. With a properly designed aircraft any roll is damped out in this way in a very short time, generally less than one second, but the machine is left with a slight list which, since the aircraft is trying to fly straight, causes it to sideslip. To correct this tendency and to restore the machine to its normal flying position is the function of the *lateral dihedral* or the arrangement of the wings in a flat vee, a device invented by Cayley and used widely ever since (Fig. 24). The dihedral, together with the sideslip velocity, causes the dipped wing to have effectively a slightly greater incidence than the raised wing. This does not affect the total lift and total drag of the aircraft but produces a rolling moment which brings the aircraft back. This correction must necessarily act rather slowly or else it would be rapidly checked, like the

ordinary roll, by the damping action of the wings. Left to itself, the aircraft executes a 'dutch roll' or relatively slow lateral oscillation as the machine overshoots and then recovers.

An aircraft resembles a ship in many ways but differs essentially with regard to rolling movements. With a ship all that matters is the position of the metacentre or the point where the line of action of the upward thrust due to buoyancy in the displaced position cuts the centre line of the ship. If this is above the centre of gravity the vessel will automatically recover from a roll, and there is no need in ships for any method of controlling the rolling motions. Perhaps the most significant contribution of the Wrights to practical flying was their realization that a positive control of roll is absolutely essential for aircraft, and although their original method, that of distorting the wing shape in order to shift the lift force from one side to another, has long been abandoned and replaced by the more convenient aileron system, essentially their ideas remain unaltered to-day.

TAILLESS AIRCRAFT

Although the tail plays an important part in maintaining the stability of a conventional aircraft it is not essential, and there are many reasons why it would be an advantage to get rid of it altogether. This is particularly so for high-speed flight, in which it is necessary to eliminate every ounce of unnecessary drag, and for this and other reasons (mainly connected with certain peculiar conditions met with in high-speed flight and dealt with in the next chapter) attention has recently been given again to an old problem, that of designing a stable tailless aircraft or its ultimate development, the *flying-wing*, in which virtually everything is eliminated except the pure lift-producing surface. Such aircraft are particularly suitable for modern methods of jet propulsion.

If the tail is eliminated, a restoring force must be provided elsewhere on the wing, and the obvious place to locate such a force is on the wing tips, provided that these can be brought well behind the centre of gravity of the whole aircraft. This is the concept behind the earliest stable tailless machine, designed by J. W. Dunne * about 1910. The wings of this machine were swept back to form an arrowhead plan-form, and control was effected by combination ailerons and elevators placed well behind the centre of gravity in order to achieve a long lever arm. Experiments were also carried out in Germany in the 1920's by Lippisch and Kohl who evolved the 'delta' design. In this country the Westland-Hill 'Pterodactyls' joined with the autogyros in delighting crowds at aerial displays in the 1930's; these were not true flying-wings and retained a fuselage beneath the wing.

The engineering problems involved in the design of a flying-wing passenger-carrying aircraft must be formidable, but there is every reason to believe that ultimately this type of design will supersede the conventional fuselage-plus-tail shape. The saving in drag obviously can be very considerable; some estimates place it as high as one-third to one-half. One thing, however, stands out very clearly; to be really worthwhile a flying-wing aircraft must be large, and for such aircraft there is a marked gain in performance by stowing all the weight in the wings,† the gain increasing with size.

At the time of writing (1948) aircraft design is in a stage of transition and little definite can be said until the many problems connected with high-speed flight have been wholly or partly resolved. These problems form the subject of the next chapter.

* Probably better known to many readers as the author of *An Experiment with Time* and other books dealing with seeing into the future.

† *v.* H. Roxbee Cox, Wilbur Wright Memorial Lecture, 1940.

CHAPTER VI

The Aerodynamical Problems of High-Speed Flight

It has already been pointed out in Chapter II that at a relatively early stage in the history of aerodynamics, ballisticians discovered that air resistance rises very rapidly when the velocity of a projectile approaches the speed of sound. At first interest in such high speeds was entirely confined to ballistics, but with the invention of the turbine it soon became a matter of necessity to study the high-velocity flow of gases through nozzles in considerable detail. In aeronautics the peculiar effects of high speeds first became evident in questions relating to propeller efficiency, but in the last few years rapid advances in the speed of aircraft have shifted the whole balance of aerodynamics to the study of what is known as *compressible flow.*

COMPRESSIBILITY

The novel features introduced by high speeds can be attributed to one property of air, its *compressibility.* This term means simply that the density of a moving element of the fluid is liable to change. An *incompressible fluid* is a mathematical idealization of a real fluid in which such density changes are excluded, and may therefore be likened to an inviscid fluid in that it ignores a characteristic property of all real gases, but in many applications the assumption of constant density can be tolerated more readily than the omission of viscosity. In a very large class of problems actual changes in density are minute, and to take account of them would mean

considerably greater complication in the mathematics with no appreciable gain in accuracy.

In an unconfined gas such as the atmosphere, large density variations can arise in several ways, the most important being: (i) the decrease of density with height, due to the great depth of the atmosphere, (ii) changes caused by local heating and cooling, and (iii) changes which occur in the neighbourhood of a body moving at high speed. The first two are the concern of the meteorologist and give rise to the branch of mathematical physics known as *dynamical meteorology*, while the last affects the aircraft engineer and the ballistician.

Flow problems in which density variations are produced or accompanied by large variations in velocity are said to belong to the science of *gas dynamics*. This is not a very happy term,* but one which has passed into common use, and is understood to imply the treatment of motion in or of gases, including thermodynamic effects. Not unnaturally, gas dynamics is considerably more difficult and less mathematically developed than either hydrodynamics or thermodynamics.

Pressure Waves in Air. If air really were incompressible we should have to admit that changes of pressure due to, say, a moving body would occur simultaneously throughout the space surrounding the body – in other words, that pressure changes are propagated with infinite speed. The actual fact is, of course, that such changes are propagated at a very high, but finite speed, which for many problems may be considered as 'infinite' without detectable error. One of the first problems to be solved in any treatment of high-speed flight is therefore a precise determination of the velocity with which a change of pressure moves through a gas. Quite apart from any question of moving bodies this is a problem which was bound to receive attention at an early stage in the history of science

* It should not be confused, for example, with the kinetic theory of gases which treats of the consequences of the molecular structure of gases.

because a sound wave is simply a succession of such pressure changes.

The investigation of the propagation of pressure variations in a gas therefore naturally starts with the problem of the travel of sound. In discussing such problems we must make one very important reservation at this stage – the magnitude of the pressure change must be very small, a condition certainly satisfied in an ordinary sound wave.* In these circumstances we can state the following result: *all small changes of pressure travel with the same velocity, which is called the speed of sound in the medium.*

The first calculation of the speed of sound was given by Newton in the *Principia*, that the velocity of propagation of a small pressure change is equal to the square root of the quotient of the pressure of the air and its density. This equation gives a result about 15 per cent. lower than the experimental value of the speed of sound. The source of error in the calculation was later pointed out by the French mathematician Laplace (1749–1827); Newton had not known that the compression of the air in a sound wave takes place so rapidly that the heat generated does not have time to get away or, in technical language, that the process is adiabatic. When a correction is made for this effect the theoretical velocity agrees closely with the best experimental values. For our purposes we may quote the result in a simple approximate form, namely

$$\text{velocity of sound waves in air} = a = 45\sqrt{T} \text{ miles per hour,}$$

where T is the absolute temperature of the air, i.e. the tem-

* The non-scientific reader may not be aware how small are the changes of pressure detected by our ears. The variations in air pressure which accompany ordinary speech are perhaps one-millionth of the normal pressure of the atmosphere, and a whisper, just detectable by a person of good hearing, would be made up of changes of pressure as small as a thousand-millionth part of one atmosphere.

perature measured in degrees centigrade from absolute zero or $-273°$, so that $15°$ C. $= 285°$ abs. and so on.

The importance of this formula for aerodynamics lies in the fact that it shows exactly how the speed of sound changes in the atmosphere, and since the temperature of the air varies with locality, height, season and time of day, we see that the speed of sound is not nearly as constant as, say, the speed of light. A representative average value for the air temperature at sea level in this country is $15°$ C. (about $60°$ F.) and from meterological records we know with fair precision the average rate of fall of temperature with height. We have then the following approximate values for the speed of sound at various heights:

Height	*Speed of Sound (approximately)*
Sea level	760 miles per hour
10,000 feet	735 ,, ,,
20,000 ,,	707 ,, ,,
30,000 ,,	679 ,, ,,

The decrease in the speed of sound with increasing height is of considerable importance in high-speed flight. The seasonal variation of temperature alone causes quite large variations in the speed of sound at sea level and this, as we shall see later, can have repercussions on attempts on speed records.

Pressure Waves caused by Moving Bodies. A disturbance causing a change of pressure in a gas is propagated as a wave consisting of a succession of condensations and rarefactions. If the disturbance is small and the wavelength lies between certain limits the result is an ordinary audible sound wave. Let us now consider what happens when a body moves through the atmosphere, first at a low speed and secondly at a speed which is comparable with that of sound, taking compressibility into account throughout.

If the reader has encountered a flock of sheep when driving

a car along a country road he will know that the flock has its own peculiar 'speed of adjustment' to the situation and that, broadly speaking, nothing will alter that speed. If the car slows down to a walking pace the flock will make way for it without a great deal of crushing, but any attempt to drive either the car or the sheep at a higher speed will end in trouble, because the flock cannot adjust its 'pattern of flow' past the car sufficiently rapidly to get out of the way. When a body moves through the atmosphere at low speed the air in its vicinity is likewise able to take up the resulting changes of pressure without much crushing together of the molecules, that is, without noticeably increasing the local density. (To put the matter in another way, when a slowly moving body arrives at a point in space the air will be ready to receive it, for any pressure changes will have been signalled ahead at the speed of sound.) If the body is moving at a velocity comparable with that of sound the density changes in its vicinity are much more pronounced, for in this case the air meeting the nose has, in a sense, insufficient time to conform to the new situation and is squeezed into a gas of higher density. In the first example the ratio of the speed of the body to the speed of propagation of the disturbances (that is, to the speed of sound) is so small that we can suppose the pressure field surrounding the body is set up instantaneously. In the second case it is obvious that we cannot make any such assumption. These arguments show that the ratio of the speed of the body to the speed of sound must be an important factor in the aerodynamics of high-speed flight, and we have here the first hint of a quantity comparable in importance with the Reynolds number.

So far the discussion has been of a general and qualitative nature. The exact analysis of the motion of a body in a compressible fluid demands a knowledge of advanced mathematical technique, but fortunately it is possible to follow the

main features from a simple geometrical treatment, provided that we restrict our arguments to small disturbances.

In Fig. 25 (*a*) we have the case of a body moving with subsonic velocity, which has reached the point A at a certain instant. As it passes A it generates a pressure wave, whose front is spherical, so that the whole disturbance grows like a soap bubble with the front itself rushing outwards at the speed of sound. At t seconds later the body has reached the point B,

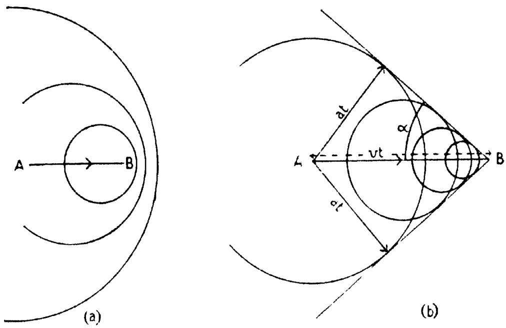

Fig. 25. Wave patterns around a moving body (a), subsonic; (b), supersonic.

so that the distance $AB = ut$, where u is the speed of the body. In this time the wave front will have travelled the distance at (where a is the speed of sound) and will therefore be ahead of the body. If we consider intermediate points we can see that each wave formed will outstrip the body and that the pattern will be as depicted, in which all the waves are contained inside the original 'bubble'. In Fig. 25 (*b*) we have a body moving at supersonic speed. Exactly as before, spherical pressure waves are formed at each point as the body passes, but because of its greater speed, the body breaks through the 'bubbles'. In this case the spherical pressure waves all lie

within the cone shown and from simple trigonometry it can be seen that the semi-angle of the cone (α) is defined by

$$\sin \alpha = at/ut = a/u$$

that is, the sharpness of the cone depends upon the ratio of the velocity of the body to that of sound.

We call the ratio u/a the *Mach number* (M) of the motion, in honour of Ernst Mach, the Austrian scientist to whom we owe many striking results and methods of studying high speed flow. The Mach number is a dimensionless quantity of the same type as the Reynolds number; if we carry out Rayleigh's analysis for aerodynamic force (Chapter III), but this time for a body moving in a compressible inviscid fluid, we find

$$\begin{aligned} \text{aerodynamic force} &= \text{number} \times \rho u^2 S \left(\frac{a}{u}\right)^n, \\ &= \tfrac{1}{2}\rho u^2 S f(M), \end{aligned}$$

where $f(M)$ can be either the drag or the lift coefficient.

This means that all phenomena in compressible flow must be related to the speed of sound, which is the 'characteristic' velocity of the medium for all these problems. We can see why the Mach number must enter into the argument if we remember that we are always dealing with an assembly of molecules; since the velocity of sound depends on the square root of the absolute temperature of the gas, which in turn is a measure of the kinetic energy of the molecules, it follows that

$$M^2 = \frac{u^2}{a^2} \propto \frac{\text{kinetic energy of the motion}}{\text{kinetic energy of the molecules}}$$

It is this ratio which largely regulates the behaviour of a gas at high speeds and in particular determines the conditions in which two high-speed flows are similar. Another way of looking at it is that M^2 represents the ratio of the inertia forces to the elastic forces in the fluid. For these reasons, and because

the speed of sound changes so considerably with different conditions, results in compressible flow are never given in terms of absolute velocities but always with reference to the Mach number.

SHOCK WAVES

In the preceding paragraphs it has been assumed that the pressure changes which accompany the motion of a body through the air are exceedingly small, but this is not always so in reality. In particular it is not true for most projectiles and for high-speed aircraft. This brings us to the deeper problem of the behaviour of large disturbances and introduces what is perhaps the most interesting feature of high-speed flow.

A complete account of the theory of pressure waves of large amplitude would involve a great deal of difficult and advanced mathematics and therefore would be out of place in a book of this type. The development of this theory is an example, not altogether uncommon in the history of science, of a phenomenon being discussed by mathematicians long before it became prominent in experimental work. Essentially, the problem reaches back to the short but memorable life of the German mathematician Bernhard Riemann (1826–66)* who in 1860 published what has since come to be regarded as the fundamental theory of waves of this type. Other notable contributions were those of Rankine (1870) and Hugoniot (1887), while in 1910 Rayleigh characteristically consolidated and materially advanced the subject in a paper entitled 'Aerial Plane Waves of Finite Amplitude'. In our own time the outstanding advances have been those made by Prandtl, Sir Geoffrey Taylor and von Kármán.

The speed of sound is, strictly speaking, the velocity of

* 'A geometer like Riemann might almost have foreseen the more important features of the actual world.' – Eddington.

propagation of *infinitesimal* pressure changes, whether audible or not. The theory of the propagation of *finite* disturbances, such as those caused by the rapid motion of a large body through the air, is naturally much more complicated, but it is possible to follow the track of the mathematics at least a part of the way without much difficulty. We can see the origin and nature of such disturbances if we think of the final velocity of a body being reached by a large number of very small increases in velocity. Each 'jerk', as it occurs, will give rise to a small pressure pulse, and if the body is moving throughout at a speed far below that of sound, the pulses will radiate from the body in all directions to form the characteristic pressure pattern. As the speed of sound is approached the situation becomes more complicated, with the local velocity exceeding that of sound at certain points, so that at this stage we can argue only on very general lines. If we have a succession of pressure pulses, each of them will accelerate slightly the fluid through which it passes while the pulse itself travels onwards at the speed of sound with respect to the fluid which lies between it and the pulse which has gone before. This means that sooner or later a pulse must inevitably catch up with that in front and so on, with the ultimate result that all the pulses pile up to form a sharp-fronted 'wave', or rather, an atmospheric discontinuity, which is rushing ahead with a speed which exceeds that of sound by an amount depending on the magnitude of the pressure rise. This is quite unlike what happens with a true sound wave whose velocity of propagation does not depend upon the loudness of the sound, that is, on the size of the pressure rise. More precisely, we can say that waves of finite amplitude, unlike those of sound, cannot be propagated without change of shape or, as the great Victorian mathematical physicist Clerk Maxwell expressed it in his *Theory of Heat*, 'In the case of very violent sounds and other disturbances of the air ... the result ... will be that if the

wave originally consists of a gradual condensation followed by a gradual rarefaction, the condensation will become more sudden and the rarefaction more gradual as the wave advances through the air, in the same way and for nearly the same reason as the waves of the sea, on coming into shallow water, become steeper in front and more gently sloping behind, till at last they curl over on the shore'.

Thus when an aircraft accelerates from a subsonic to a supersonic speed the finite pressure waves which it generates join together to form a single steep-fronted wave travelling with a velocity rather greater than that of sound. Such a wave cannot get very far away from the body because as it moves it rapidly loses energy and degenerates into a sound wave, which is caught up by the body and carried along with it.

Pressure fronts of this type are called *shock waves* or *compression shocks* and the term is understood to mean a well-defined surface or wave-front across which velocity, pressure and density undergo very sharp changes. A shock wave is essentially a region of discontinuity; Rayleigh used the term 'aerial bore', having in mind the phenomenon seen in tidal rivers such as the Severn. With a body such as a high-velocity shell there is a large, well-defined shock wave at the nose merging into the Mach cone at points far away from the body. Other shocks spring from irregularities on the surface, principally the driving band *, and also at the rear, where the flow is bent back.

It has been proved mathematically that all finite disturbances of the type formed by rapidly moving bodies can exist only for a brief time before forming a shock wave and also that the surface of discontinuity thus created is extremely thin.

* The driving band of a shell is a ring of soft metal set into the body and protruding above it. The rifling in the barrel engages with the band and cuts deep grooves in it, so that a gas-tight seal is formed and the shell is given the necessary spin (because of the helical form of the rifling) without undue wear of the gun itself.

In the case of a shell travelling much faster than sound, for example, it has been calculated that the entire change takes place in a layer of air which cannot be more than about a ten-thousandth of an inch in thickness. This explains why it is possible to photograph shock waves relatively easily, by taking advantage of the fact that the change of density must be extremely sharp in such a thin layer. There are several methods in use, the simplest being to arrange for a powerful electric spark to cast a shadow of the bullet as it passes in front of a photographic plate. If shock waves are present the change of density causes the light from the spark to be refracted or bent on passing through the discontinuity, so that it does not reach the plate. In this way any shock waves appear as thin black lines on the print, rather like cracks on a sheet of plate glass (Plate 5). During the bombardment of London by V-weapons in 1945 many people undoubtedly saw the shock waves caused by distant explosions rising from the ground like gigantic bubbles.

When a high-velocity shell passes overhead three distinct sounds may be heard. The first is a sharp intense crack, marking the passage of the shock wave attached to the nose of the shell. This is followed by the deeper and more prolonged report of the gun itself, that is, the remains of the shock wave generated by the propellant gases expanding from the muzzle. Finally, there is the sound of the shell exploding, which may precede or follow that of the gun. If aircraft ever reach speeds comparable with those of projectiles we shall probably grow accustomed and indifferent to the intense crack of a supersonic machine darting across the sky, even as our grandfathers were to the crack of a carter's whip, for that also was a shock wave, caused by the tip of the lash moving through the air at a speed greater than that of sound.

The Anatomy of a Shock Wave. Fig. 26 shows what happens when a supersonic stream of air strikes a blunt-nosed

body (or equally, when such a body is moving through the air at supersonic speed). The effect of the blunt nose is that

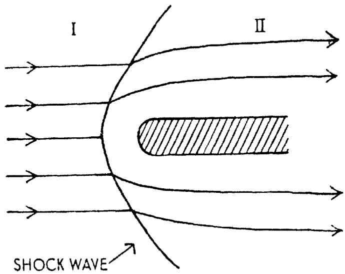

Fig. 26. Shock wave at the nose of a blunt body moving faster than sound.

the shock wave is formed some distance in front and the true Mach cone is only found well out to the side. In the region marked I the air is moving (relative to the body) faster than sound, and this is a region of low pressure. At the shock wave itself the speed is reduced in passing through the discontinuity, so that between the wave and the nose (region II) the air is moving (relative to the body) at less than the velocity of sound. This is a region of high pressure through which any disturbances set up by the body can be propagated forwards, eventually to pile up on the shock wave. These features have been verified by measurements made in wind tunnels.

LIFT AND DRAG AT HIGH SPEEDS

The large increase in resistance experienced as a body approaches the speed of sound has now become common knowledge. This aspect of the problem of high-speed flight has been prominent recently in the non-scientific press and it is to be feared that on occasion not a little nonsense has been written about 'penetrating the barrier of sound' and the like.

There is no difficulty whatever in making a body travel faster than sound – gunners have been doing it for years. If supersonic flight simply meant making a unit powerful enough to push a flying-machine through the air at the requisite speed the problem would at least be straightforward, although not easy even with present-day resources. The real task – to design a piloted aircraft which can take off, fly at supersonic speed and then land safely – is much more difficult and so far a solution has not yet been demonstrated, although scientists generally believe that piloted supersonic flight can ultimately be achieved. To appreciate the advances now being made we must first see what the difficulties are, why they arise and what methods are available to overcome them.

Drag. All the early knowledge of resistance at high speeds arose from gunnery; after Robins and his ballistic pendulum (1740) came Hutton (1775) and Didion (1840). All of these used round shot. The earliest experiments with pointed shell seem to have been those which the Rev. Francis Bashforth, Professor of Mathematics in what is now the Military College of Science, carried out between 1865 and 1870. Bashforth discarded the ballistic pendulum and, by means of an electrical chronograph of his own invention *, he determined very accurately the times at which the shell passed through a series of screens and thus found the velocity along the trajectory and finally the resistance law. In this way he was able to construct resistance-tables for a standard shape of head for velocities between 100 and 3,000 feet per second (70 to 2,000 miles per hour approximately). These data were used in the calculation of British range-tables until 1910. Bashforth's lead was followed on the Continent and at the end of the nineteenth century most countries had their own laws of resistance suitable for their own shells. In Germany, Krupp and Cranz did

* This apparatus is still in excellent working order (a tribute to Victorian workmanship) and was shown at the Physical Society's Exhibition in 1948.

valuable work of a similar nature, the latter producing the classical book on the subject. The full significance of this work did not immediately become apparent. Much of ballistics constitutes valuable military secrets which governments are naturally reluctant to release, and further, ballisticians have always had their own peculiar ways of expressing their results, usually in a form designed to facilitate the computation of range-tables, but which take a little unravelling if it is desired

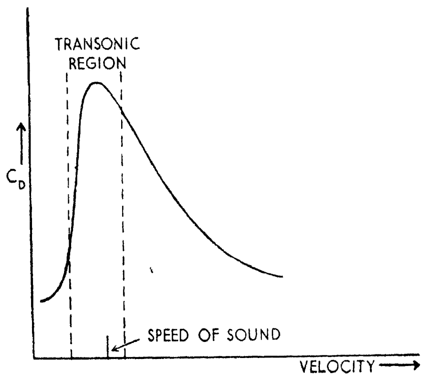

Fig. 27. The drag coefficient of a projectile (not to scale).

to make comparison with normal fluid mechanics. It is only in recent years that ballistics has really started to join up with aerodynamics.

Leaving aside all matters of detail, the general behaviour of the drag coefficient of a pointed projectile at high speeds is that shown in Fig. 27, where C_D is plotted against the Mach number M. (At these high speeds viscosity, and hence the Reynolds number, has relatively little influence.) The most striking and obvious feature is the very sudden rise in C_D just before the velocity of sound is reached. After the speed of

sound is passed the drag coefficient starts to fall again, but the actual resistance, being proportional to the product of the drag coefficient and the square of the speed, still rises as the velocity increases. This sudden rise is the famous 'barrier of the speed of sound'; at this point the Newtonian or 'velocity squared' law ceases to have any real significance, although for convenience we still use it in the definition of C_D.

The sudden increase in the drag coefficient indicates the extra energy demanded and extracted from the motion of the shell to form the shock waves, so that most of the resistance now arises from causes utterly unlike those which obtain at moderate speeds. At the lower speeds the greater part of the energy used in overcoming drag is dissipated in eddies in the wake and at such speeds the shape of the tail is rather more important than that of the head. At high speeds the position is completely reversed, for in these conditions the greatest loss of energy occurs in the pressure shocks around the head of the shell, and for high efficiency the shape of the head is more important than that of the tail. (This is fortunate for gunnery, because it would be extremely difficult to design a practical shell with a completely streamlined body.)

A similar effect is found with aerofoils, the onset of the rapid rise in drag varying somewhat with the shape of the aerofoil and with the angle of incidence. As a general rule the drag coefficient at any fixed incidence remains approximately constant until the Mach number reaches about 0·6, after which there is a very rapid rise, the drag coefficient increasing in value between five and ten times in the relatively narrow range of speeds whose Mach number lies between 0·6 and 0·8. The greater the angle of incidence, the lower is the Mach number at which the upward jump starts.

These facts can all be summarized in the statement that on approaching the speed of sound there appears an added resistance which has no counterpart at moderate speeds, and

which can be attributed to the energy radiated from the body in shock waves. For this reason the additional resistance is usually called *wave drag*. The picture of resistance as a whole becomes clearer if we think of the analogy with the cooling of a hot body in a stream of air. Such a body loses heat (that is, energy) in three ways, by conduction, convection and radiation. Conduction is the transport of heat from the surface by molecular agitation and therefore corresponds exactly to skin friction, which is the conduction of momentum away from the moving body by viscosity. Convection is the loss of heat from the body caused by the breaking away of relatively large masses of heated air, and this is very much like the loss of energy suffered by a moving body because of the washing away of eddies in the wake. Radiation means a loss of heat by wave motion and is of importance only at fairly high temperatures, thus resembling wave drag which becomes appreciable only when the speed of the body nears that of sound. The analogy in all these cases is more than superficial and extends into the mathematical treatment.

Lift. A large increase in resistance on approaching the speed of sound implies, of itself, considerable difficulties in design but, unfortunately, this is not the only adverse effect encountered in this region of speed. There is a corresponding change in the lift force and this is far more serious and difficult to counter.

In 1927 the English mathematician H. Glauert, whose untimely death in an accident in 1934 was a particularly severe blow to science, evolved a simple rule to allow for the effect of compressibility on the lift of an aerofoil, and in 1930 Prandtl independently discovered and published the same formula, which is now known generally as the *Glauert-Prandtl rule*. This affords a means of deducing the behaviour of an aerofoil in compressible flow from its known characteristics in incompressible flow and may be stated thus: the forces on an aerofoil

at high speed can be obtained from those found at moderate speed by multiplying by the factor $1/\sqrt{1-M^2}$ where M, as usual, denotes the Mach number. In particular the lift coefficient of an aerofoil should be increased by compressibility in the ratio $1/\sqrt{1-M^2}$.

It is evident that as M approaches unity, that is, as the speed of the air over the wing nears the velocity of sound, the rule must break down, because it leads to the impossible result that the lift of a wing would be infinite at the speed of sound. This much was obvious from the start; what was not known was how far the formula could be relied upon in the scale of ascending velocities.

An answer was first given by the British scientists G. P. Douglas and R. McKinnon Wood in 1922 from their researches on high-speed airscrews. One of the easier ways of getting really high-speed air flow over an aerofoil shaped section is to speed up a propeller until the tips of the blades are moving at velocities comparable with that of sound. Douglas and Wood, and later Douglas and W. G. Perring, found that the lift coefficient increased, as predicted, when compressibility effects started to appear, but when they raised the propeller speed still further to correspond to Mach numbers between 0·6 and 0·7 they observed a sudden drop in lift and a sharp rise in drag. (This loss of efficiency as the tip speed nears that of sound is mainly responsible for the necessity of using jet propulsion in very fast aircraft.) A little later Sir Thomas Stanton extended and verified these results by tests on an aerofoil section in a special high-speed wind tunnel; his curves show that lift increases with speed up to about $M = 0{\cdot}7$, after which there is a sudden and catastrophic fall. The higher the incidence, the lower is the Mach number at which the discontinuity appears.

Shock Stall. These observations indicate one of the main difficulties and dangers encountered in high-speed flight. At

about three-quarters of the way up to the speed of sound the lift/drag ratio of an ordinary wing collapses, exactly as in the case of an aerofoil pushed up at a steep angle of attack. In other words, the wing stalls and no longer shows the characteristic features of a true lifting surface. This is the phenomenon known as *shock stall*, and it is primarily due to the fact that at some point on the surface of the aerofoil the local speed of sound has been reached, resulting in the formation of a shock wave perpendicular to the surface. As in the case of stall at moderate speed, the loss of lift is chiefly caused by separation. Behind a shock wave is a region of high pressure, so that on reaching this point the boundary layer flow is brought up against the retarding effect of a strong adverse pressure gradient, which ultimately blocks its movement to the rear and causes it to separate and form a much wider wake. This suggests that one way of alleviating shock stall is boundary layer control, and much research has already been directed to this end.

The Mach number at which the above effects of compressibility start to appear is known as the *critical Mach number* and it marks the beginning of what is called the *transonic region of flow*. Some writers define the critical Mach number as the free stream value of M for which the local speed of sound is reached at some point on the aerofoil, but this is a somewhat theoretical definition, not too easy to use in practical work. The catastrophic effects referred to above, which give rise to shock stall, first definitely appear at a slightly higher value of M than that required to reach the velocity of sound at some place on the aerofoil, because the piling up of the disturbances becomes sufficiently intense to halt the downstream motion of the boundary layer and force it off the surface only when the higher value is reached.

The transonic region cannot be rigidly defined – usually it is considered to be a region lying somewhere between

$M = 0{\cdot}75$ and $M = 1{\cdot}2$, of mixed subsonic and supersonic motion, rather like the transition zone between laminar and fully turbulent flow, and therefore very difficult to analyse mathematically. At present the indications are that the transonic region is likely to be by far the most difficult and dangerous zone for the control of an aircraft and that in the true supersonic region problems will become easier in this respect. One might say, in fact, that at the present time the difficulty of control in the transonic region is the real barrier to supersonic flight.

THE DESIGN PROBLEMS OF HIGH-SPEED AIRCRAFT

Since supersonic flight can only be reached by passing through the transonic region, it is evident that a great deal of research into conditions which obtain around the speed of sound is called for. This is difficult for two reasons. One is that the mathematical theory of motion which is partly subsonic and partly supersonic must of its very nature be complex and, in fact, hardly exists at the present time. The second is that wind tunnel tests covering this band of velocities are extremely difficult to carry out, for a variety of technical reasons which need not concern us here. However, the rapid development during the past few years of the technique known as *telemetering*, or the use of electronic and radio aids to make 'robot' measurements in unmanned test-aircraft and to transmit the readings to the ground, has made free flight experiments possible without risking valuable lives, and it is to be expected that this approach will be used more and more in the future. Compared with wind tunnel work it is, however, expensive in material and time, but against this can be set the high costs of the design, erection and maintenance of high-speed tunnels. Whichever way is adopted, the attainment of supersonic flight can be effected only on a national scale, and

the days of pioneers like the Wright brothers, who built their machine in a cycle repair shop, are gone for ever.

Meanwhile a considerable amount of advance information is available from the existing mathematical theory and the results of wind tunnel tests. The first thing, as Kármán noted in his Wright Brothers' Lecture to the Institute of Aeronautical Sciences in 1947, is that 'the aeronautical engineer should get the same feeling for the facts of supersonic flight as he acquired in the domain of subsonic velocities' and this often means a radical change in his basic conceptions, for what is 'right' at subsonic flow may be entirely wrong for supersonic flight.

In the first place, every device possible for reducing wave drag must be exploited to the limit, because to attain supersonic speed in a manned aircraft is bound to demand extremely powerful and efficient power units. A calculation made in 1946 * on the power required to fly a theoretical supersonic aircraft, designed for the lowest possible resistance with the aid of data obtained on projectiles, shows how formidable the problem is. The design in question is for an aircraft with a long pointed nose and streamlined tail, of about 4 feet maximum diameter, to fly at a Mach number of 1·5 at 50,000 feet, i.e. in a very attenuated atmosphere. To do this requires over 2,000 horse-power, but to reach the same Mach number at sea level, where the air resistance is much higher, would need over 20,000 horse-power. It is an open question whether it will ever be possible to fly faster than sound in the lower levels of the atmosphere (which may come as a relief to many people) and there does not appear at present any very good reason why anyone should want to do so.

To reduce wave drag to a minimum the nose of the fuselage must be long and pointed – a conical shape suggests itself here.

* A. C. Charters, 'Some Ballistic Contributions to Aerodynamics'. *Journal of the Aeronautical Sciences.*

The wings will probably differ considerably from those employed on low-speed aircraft. The wave drag is proportional to the square of the thickness/chord ratio and the induced drag at supersonic speeds involves the square of the lift coefficient, so that the wings of a supersonic aircraft should be thin and of low lift.

At supersonic speeds the effect of camber is not evident and there are no aspect ratio effects, so that the designer can use wings of low aspect ratio, should he so desire, without reducing lift.

In addition to the appearance of shock stall, entry into the transonic region means that the flow around the tail-planes is affected by disturbances in the wake of the wing. The decrease in circulation means diminished downwash, resulting in a tendency for the machine to lose trim and dive. This is one of the major dangers in transonic flight.

Methods of overcoming Compressibility Effects. Research into ways of delaying, if not overcoming, the adverse effects of compressibility is going on at present in aeronautical institutions all over the world, with the result that many unusual designs have been suggested. The difficulty with tail-plane control surfaces, for example, would be removed if the machine flew 'tail first', and although such an aircraft might appear extremely odd to our eyes, there is no reason why it should not function quite well.

We shall consider here only one of the many suggestions which have been advanced, namely that of the swept-back wing (Fig. 28). There is nothing unusual about the appearance of an aircraft with swept-back wings, for this arrangement has been used in many types of subsonic aircraft, with the wing either directed backwards or with a receding leading edge. At moderate speeds sweepback serves to displace the aerodynamic centre towards the rear, thus improving stability, and it has the further beneficial property

of increasing course stability, which means that if the aircraft is forced out of its course by a disturbance the swept-back wing arrangement tends to make it return of its own accord.

In high-speed flight the use of the swept-back wing is favoured for an entirely different reason, originally suggested by the German scientist Busemann at an international conference on high-speed flight held in Rome in 1935. Busemann's scheme may be explained thus: if a means could be found of raising the critical Mach number of a wing, the onset of shock stall would be delayed. This result can be obtained by arranging the geometry of the wing so that a lower velocity is obtained perpendicular to the leading edge, because this is the velocity which determines the critical Mach number, and reference to Fig. 28 shows that if the wing is swept back through an angle θ, the component of the speed of the airflow over the wing in the direction perpendicular to the leading edge is reduced to $v \cos \theta$, where v is the speed of the aircraft. That is, the effective critical Mach number for the wing is increased by the factor $1/\cos \theta$, or, in other words, the aircraft can fly at a speed higher than that which would produce shock stall conditions over a straight wing. This very simple but ingenious idea did not receive the attention it deserved at the time it was suggested.

The arguments given above refer strictly to a wing which has an infinitely long span, and when measurements were made on a wing of finite span it was found that the benefit is not as great as that indicated by the simple theory, the true factor being nearer to $1/\sqrt{\cos \theta}$. There are also other drawbacks in practice, but opinion seems generally to incline to the view that the net effect of sweepback is beneficial. It may be noted that the arguments also apply to sweepforward, but for other reasons (chiefly stability) this arrangement is not likely to be used in high-speed aircraft. An attractive variant of the sweep-

back idea is the so-called 'delta' or triangular wing aircraft of the flying-wing variety.

At the time of writing (1948) events are moving so rapidly in the world of aeronautical science that it would be unwise to venture any detailed prophecy concerning the shape, size and general features of the first aircraft to take off, fly at supersonic speed and land safely. As a guess it would be rocket-

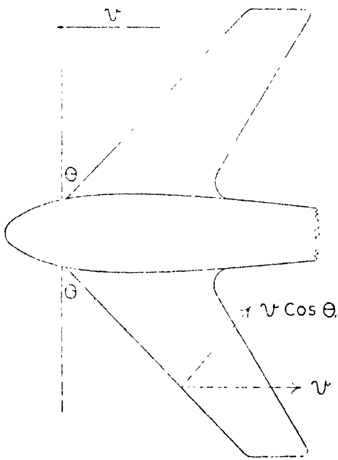

Fig. 28. Aircraft with swept-back wings.

propelled, because as yet only this type of propulsion can supply sufficient energy (and even then for a short time only, because of the considerable weight of fuel consumed). Nor does it seem probable that any of the very unorthodox shapes will be used in the first attempts – instead the machine is likely to have a body with a long tapered nose and short, thin, but very strong wings, probably swept-back. The only thing that seems quite certain is that it will be designed primarily to

operate at very great heights, in the intensely cold and cloudless rarefied air of the stratosphere.

AERODYNAMIC HEATING AT HIGH SPEEDS

We conclude this short account of high-speed flight and its peculiarities with a brief mention of a feature which is likely to become of increasing importance as the speed of aircraft increases. This is the heat developed by the passage of the machine through the air.

How this effect arises can be best understood by thinking of the equivalent problem of what happens when a moderately high-speed air stream strikes a stationary rounded object, such as a thermometer bulb. At the extreme forward point the air is brought to rest, but elsewhere it flows around the obstacle with varying speed. The point at which the air stream is halted is called, very appropriately, the *stagnation point* (it may, of course, be a line) and it is here that the pressure reaches its highest value. Air in motion has kinetic energy, measured by the product of its density and the square of the speed, and when the stream is brought to rest this energy, since it cannot be destroyed, must be changed into another form. It is transformed into heat and thus there is a rise of air temperature at the nose of the obstacle.

The magnitude of this rise is easily calculated if it is assumed that all the energy of the motion is immediately converted into heat. By an accident of figures the expression assumes a particularly simple approximate form when the speed of the air stream or of the moving body (u) is expressed in miles per hour. This is

$$\text{rise of air temperature in degrees centigrade} = (u/100)^2$$

or, in other words, the rise of air temperature is equal to the square of the speed of the body when this is expressed in

hundreds of miles per hour. Thus even at what are now regarded as quite moderate speeds, the rise of temperature is very marked – at 250 miles per hour, for example, the stagnation point temperature is increased above that of the ambient air by $(2\cdot5)^2 = 6\frac{1}{4}^\circ$ C. $= 11\frac{1}{4}^\circ$ F. At higher speeds, such as those attained by jet-propelled fighter aircraft, the rise is even more pronounced; thus at 500 miles per hour the increase is 25° C. $= 45^\circ$ F., which is rather more than the difference between average winter and summer temperatures in this country.

The exact stagnation point temperature is not reached at all points on the body, since it is only at the extreme forward point that the whole of the kinetic energy of the motion is converted into heat. In practice it is found that the rise recorded by a well-exposed thermometer varies somewhat with the shape of the thermometer element, and a correction factor has to be used to qualify the square of the speed term. These considerations have important consequences in observational meteorology, since the temperature of the upper air is one of the major factors in weather forecasting. Such temperatures can be determined in one of two ways, by sending up small balloons carrying instruments whose indications are transmitted to earth by radio, or by mounting distant-reading thermometers (generally of the electrical-resistance type) on the wings of aircraft and taking readings during a flight. A sounding-balloon rises so slowly that the speed correction is negligible compared with other unavoidable errors, but this is very far from being true with modern high-flying aircraft. Ideally, the meteorologist would like to know the temperature of the air fairly accurately, certainly within about half a degree Fahrenheit, and this means that very considerable care has to be taken to evaluate the correction factor, for what the observer records may be as much as 10° F. or more above the true air temperature.

The highest speed yet attained by any man-made object in flight through the atmosphere is probably that reached by the giant V2 rockets which the Germans launched against London in the closing stages of the war. This was of the order of 3,000 miles per hour in the final stage of the trajectory, which means that the sharp-pointed nose of the rocket must have been at about 1,000° C., or white-hot, and it has been estimated that the casing of the rocket attained about 300° C. during the downward plunge.* Stories of observers seeing V2's glowing red in the skies may thus be true.

Meteorites, or shooting stars, are small bodies which enter the atmosphere at extremely high speeds, perhaps 45,000 miles per hour. Despite their very small size – the average meteorite is only a fraction of an inch in diameter – they leave a brilliantly luminous trail, visible through many miles of air. For meteorities the simple compression theory would indicate a stagnation point temperature of about 200,000° C., but this is certainly an over-estimate, since the effects of radiation cannot be neglected, and the whole problem is extremely complicated. It is not difficult to see, however, why a small body moving at such speed quickly burns away.

* Kooy and Uytenbogaart, *Ballistics of the Future* (1946).

[*Postscript.* The above account of high-speed flight was written in 1948. When this book was being set up, it was announced that piloted supersonic flight had been achieved in America. This presumably means that the problem of control in the transonic region is now well on the way to being completely solved, but as yet no details are available.]

CHAPTER VII

The Ultimate Flying-Machine

THE closing stages of the war brought to the world, and to the inhabitants of Southern England in particular, the realization that attacks from the air had suddenly assumed a new and exceedingly menacing form. England was being bombarded by projectiles of a size hitherto unheard of, covering enormous ranges and rising to incredible heights, ultimately crashing to earth at a speed which far exceeded that of an ordinary shell. It quickly became known that these projectiles were *rockets*, an ancient device, but now developed by German patience and ingenuity beyond anything hitherto thought practicable. Since the close of hostilities the potentialities of rocket flight thus revealed have aroused intense interest in all countries, and even travel beyond the limits of the atmosphere is no longer regarded as utterly fantastic, although certainly very far from being realized with present-day resources.

Jet propulsion of ordinary aircraft is now well established, but the true rocket is unique among all devices for overcoming gravity in that it does not rely in any way on the air to sustain its motion, and in this sense the rocket must be regarded as the ultimate flying-machine. Whether or not interplanetary travel can ever be achieved is unknown, but it is clear that nothing but a rocket offers a chance of escaping from the earth's gravitational field, because no other means of propulsion can hope to attain the necessary high velocity in the attenuated upper atmosphere and in the empty space beyond.

THE HISTORY OF ROCKETS

It seems reasonably certain that rockets were invented by the Chinese not later than the thirteenth century A.D.; there is a rather obscure reference to 'arrows of flying fire' in an account of the repulse of Mongol invaders at the siege of Pien-king in 1232. The discovery of the principle of reaction which gives the rocket its thrust goes back much further, at least to Hero of Alexandria (*c.* B.C. 300) who is credited with the invention of the first steam engine, a toy which rotated by the reaction due to two jets, and there is some reason to believe that the mechanical flying bird of the philosopher Archytas (*c.* B.C. 360) was really a kind of suspended rocket, probably again using steam. Whatever may be the real place and time of discovery of rockets, it is certain that the secret did not remain very long in the Far East, for a few years after the date of the battle of Pien-king there is an account of a gunpowder rocket, called a 'Chinese arrow', in an Arab manuscript, and European chronicles of the thirteenth and fourteenth centuries also refer to their use in war. The sketch-book of an Italian military engineer, Joannes de Fontana (*c.* 1400) shows that there is nothing new under the sun, for it contains drawings of a rocket car and of a rather oriental-looking rocket torpedo, but there is no definite record of either having been used.

From China also comes the story of the first attempt to achieve a rocket-propelled man-carrying flying-machine. About 1500 (the date is uncertain) a certain mandarin named Wan-Hu placed himself in a chair to which was attached two large kites and forty-seven gunpowder rockets. At a given signal forty-seven coolies lit the rockets, but exactly what happened after this is obscure, except that the 'flight' was certainly short and presumably disastrous to the mandarin.

In military history the rocket has made several sporadic and

generally short-lived appearances. Crude gunpowder rockets, stabilized by long bamboo poles, were used by the Indian troops of Hydar Ali and his son Tipu Sahib against the British Army in the campaign of 1780–4 and at the siege of Seringapatam, apparently with considerable effect. The first systematic investigation of military rockets and their development into real weapons of war is due to Sir William Congreve in the opening years of the nineteenth century, when Britain was engaged in a life-and-death struggle with Napoleon. These rockets were used with great effect in an attack on Boulogne in 1806 and against Copenhagen in 1807. During the latter battle no less than 25,000 rockets were fired and the city was almost completely destroyed by fire and blast.

The rockets designed by Congreve had all-up weights varying from 12 lb. to 42 lb., and carried war-heads weighing up to 18 lb. They achieved ranges from 2,000 to 3,000 yards, but were very inaccurate and therefore of use only for the mass bombardment of targets covering considerable areas. Their success in this role brought about the formation of rocket corps in most European armies, but by the second half of the nineteenth century nearly all of these had reverted to conventional artillery units, for in the meantime the rifled gun and the pointed projectile had been invented, and these had introduced into gunnery new standards of accuracy which made rockets obsolete. After that the rocket survived in two forms only, as a firework intended for amusement and in the coastal life-saving apparatus.

So much for military applications; meanwhile the rocket seemed to exercise a powerful attraction for a host of inventors, some of whom were concerned with the propulsion of airships while others dreamed of entirely mechanical flight. The story of these early attempts is entertaining, but little else, and it is not until after the war of 1914–18 that the modern rocket begins to emerge. In 1919 the Smithsonian Institute of

America published a paper by R. H. Goddard containing suggestions for the exploration of the upper atmosphere by means of instrument-carrying rockets, but for some unexplained reason the real centre of activity was Germany, where a group of enthusiastic amateurs seized on the idea of interplanetary travel with avidity and soon became busily occupied in designing and constructing rockets of a novel type. One of this group, a young teacher of mathematics named Hermann Oberth (b. 1894), published in 1923, more or less at his own expense, a small paper-covered pamphlet entitled *Die Rakete zu den Planetenraumen* (The Rocket in Interplanetary Space). This book, and its successor *Wege zur Raumschiffahrt* (The Way to Space-Travel), published in 1929, are important for two reasons. First, they were written by a competent mathematician and not by an imaginative novelist or a seeker after notoriety, and secondly, they gave for the first time a correct quantitative analysis of the mode of action of a rocket and of the features necessary for sustained high-speed flight. The main conclusion of the books, that it is possible to escape from the earth's gravitational field with the aid of a properly designed rocket, is more sensational but really less important, for at this stage no account could possibly be taken of the stupendous engineering problems on which the practicability of such a feat really turns. Oberth's analysis showed that interplanetary travel is not theoretically impossible, but this is very different from saying that it can ever be achieved.

The period between 1923 and 1936 is one of confused and largely uncoordinated activity in Germany, consisting mainly of attempts by ardent but not particularly well-informed amateurs to evolve practical long flying rockets. An account of German work in this period, based on inside knowledge, has been given by Willy Ley, one of the pioneer workers in this field, in his book *Rockets: The Future of Travel*

Beyond the Stratosphere (New York, 1945). Many of the efforts were little more than stunts and some were completely ludicrous, such as the 'Magdeburg Project' (1933), which, according to Ley, originated in a desire to test a preposterous theory that the true shape of the universe is a hollow globe, but even this has its points of interest since it shows that at this date some Germans were thinking seriously of a monster rocket, about 25 feet long, shaped like an artillery shell, which is something not unlike that which ultimately emerged as V2. It was during this period that the *Verein für Raumschiffahrt* (Society for Space-Travel) was founded and a monthly magazine *Die Rakete* (The Rocket) published. There were other activities not connected with flying: Fritz von Opel, maker of the popular German car of that name, became interested and some not very impressive runs were made with the Opel Rocket Car. This must be regarded as something in the nature of a diversion, for the efficiency of such a vehicle is exceedingly low, probably not more than a few per cent, and a gunpowder rocket-car can hardly be taken seriously.

From the welter of failures, personal jealousies, squabbles and heroic efforts chronicled by Ley two facts emerge; the first is that gunpowder, the traditional rocket fuel, is completely unsuitable for large long-burning rockets, and secondly, that the real future for rockets lay in the development of liquid fuel systems. The first conclusion need cause no surprise, for anyone who has worked with explosives knows that gunpowder, even in the hands of experts, is much more difficult and dangerous to use than colloidal propellants such as cordite, but the second conclusion marks an important advance.

The liquid fuels suitable for rockets are not numerous. They may either be single liquids of an explosive nature and therefore dangerous to handle, or else combinations of liquids which react violently and produce large amounts of energy at

high temperatures. (An example of the second type is the alcohol-liquid oxygen reaction used in V2.) The problems which arise in using such fuels in rockets are mainly of an engineering nature, but it is engineering of a high order and of an unusual kind, the technique of which has to be acquired by patient and often hazardous research. Such problems were entirely beyond the capacities of the rather happy-go-lucky amateurs who first entered this field, and it is no surprise that this period is marked by a monotonous succession of failures, not of the principles, but of the rockets themselves, which almost invariably exploded or disintegrated in the first few seconds because they were not properly designed to withstand the enormous stresses developed.

About 1935 it seems to have occurred to some unusually far-sighted Nazi official that there might be something, after all, in the large claims made by the enthusiasts. The result was typical. The *Verein für Raumschiffahrt* ceased to exist and its journals, records and equipment fell into the hands of the Gestapo; the mechanics who worked at its modest *Raketenflugplatz* or rocket-testing ground near Berlin disappeared into industry, and the Press was informed that the word 'rocket' and all its associations were *verboten*. The rest of the story did not become known until after the German surrender in 1945, when the truly amazing advances made at Peenemunde and elsewhere were revealed. The professionals had taken over and had made the long-range rocket a fact.

In this country rocket research on a large scale was not started until a few years before the outbreak of war, when Sir Alwyn Crow headed a team of workers mainly drawn from the Research Department, Woolwich Arsenal. This group succeeded in solving the problem of the design of relatively small but powerful high-speed solid fuel (cordite) rockets for anti-aircraft defence and other purposes. On the purely aircraft side Air Commodore Sir Frank Whittle and

others quietly proceeded with the development of the gas turbine, one application of which is to the jet-propelled aeroplane which, at the time of writing (1948), has made the nearest approach as yet to the velocity of sound. This, since it requires an atmosphere for sustenation, is an aircraft and should not be confused with the true rocket, which needs no such aids and actually works most efficiently in empty space, but both rely on the same mechanical principle for their motion. It is this principle, so often misunderstood, which will be considered in the next section.

THE PRINCIPLE OF THE ROCKET

In its simplest form a rocket is a tube closed at one end containing a substance which on ignition produces very rapidly a large amount of gas, which rushes out of the open end and forms a high-speed jet. In small rockets, such as those used in firework displays or to supplement artillery in war, it is customary to use a solid fuel, such as gunpowder, or one of the colloidal propellants, such as cordite, used by modern artillery. With military rockets the pressure generated inside the tube usually lies between 1,000 and 2,000 lb. per square inch, and the velocity of the gas at the orifice is about 4,000 miles per hour, or some six times the velocity of sound at normal temperatures. The time of burning is usually quite short, perhaps a second or two. For ordinary fireworks the pressures are naturally much less. Large rockets, on the other hand, use liquid fuels and work with very much lower combustion chamber pressures, generally a few hundred pounds per square inch, but they have very much the same jet velocities as the smaller types. The time of burning of such rockets is very much longer than for the usual types of smaller rockets; with V2 it was of the order of a minute.

One of the most widespread fallacies about a rocket is that

it works by pushing against the air behind it. This is easily refuted, but another, that the whole rocket principle is summed up in Newton's Third Law of Motion ('Action and reaction are equal and opposite') is more difficult to counter because it embodies part of the truth. The Third Law asserts that an action in one part of a system cannot be isolated but must have immediate consequences elsewhere, but the principle which is essential in rocket theory is that contained in the Second Law of Motion, which may be expressed in the form: *the rate of change of the total momentum of a system is equal to the sum of the external forces acting on the system.*

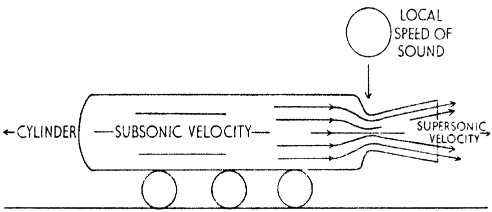

Fig. 29. The elementary principle of the rocket.

Momentum, originally called by Newton the 'quantity of motion', is defined as the product of mass and velocity and is the precise expression of the idea we are trying to convey when we speak of a 'heavy blow', meaning the effect of an impact of a body which is both heavy and fast. These are rather difficult and abstract conceptions which can be made easier by considering some concrete examples.

The Qualitative Explanation of Rocket Action. Suppose we have a cylinder containing air at high pressure which is placed upon rollers (Fig. 29). The whole system, consisting of the cylinder, its contents and its supports, is then in equilibrium. The weight of the cylinder, that is, the force due to gravity, is exactly balanced by the reaction of the rollers, and the

internal pressure, or the force exerted by the molecules of air on the walls of the cylinder, is likewise exactly balanced by the forces due to the molecules which make up the steel walls.

Suppose now that one end is suddenly opened, so that the air imprisoned within the cylinder rushes out at a speed which, for reasons which will be explained later, we may safely assume to be well above the velocity of sound. Part of the system has now acquired momentum, although no external forces have been applied. By the Second Law of Motion this means that the momentum of the system *as a whole* has not changed, so that somewhere else in the system an equal and opposite amount of momentum must appear to balance that of the emerging air. This can only mean that the cylinder itself must move along the rollers in a direction opposed to that of the air particles in the jet. If the cylinder be fastened down so that it cannot move, an external force is called into being, exactly equal to the rate of change of the momentum of the air, and this is the thrust developed by the rocket action. (This illustrates how the thrust of a rocket is measured in the laboratory by placing the rocket on rollers and measuring the force required to prevent it from moving when the charge is fired.)

We can also think of the matter more intuitively as follows. Before the air is released the pressures on the forward and rear ends exactly balance, like the two pans of a scale when carrying equal weights. When the pressure at one end of the cylinder is released, the effect is exactly the same as that of lifting one of the weights from a pan so that the other pan descends, that is, moves in a direction opposite to that of the weight removed. In the rocket the unbalanced weight is the pressure on the forward end, which is now left free to push the container along.

It should be noted that in the above no account has been taken of conditions outside the rocket, and it might be argued

that, in considering the system as a whole, it is impossible to ignore the fact that the jet has to force its way into the surrounding atmosphere. This is perfectly true in general, but in this special case the jet is moving with a highly supersonic velocity and this prevents the effects of outside conditions ever reaching the rocket. In Chapter VI it was explained that disturbances in a fluid, if they are small, move with the velocity of sound; this means that no such external pressure change can fight its way up the jet to affect the rocket itself. The exact conditions in which this statement is true will be considered later; meanwhile it suffices to say at this stage that these conditions are invariably satisfied in a practical rocket.

The balance referred to is not between velocities but between momenta, and accordingly the massive cylinder moves far more slowly than the jet of air. The force which drives the cylinder along the rollers is exactly the same as that which makes a cannon recoil. The cannon ejects a relatively small mass (the shot) at very high speed and balances the momentum by moving backwards itself, but since the mass of the cannon is far greater than that of the shot, its velocity of recoil is far lower. A man on skates could equally well cause himself to move backwards by throwing a heavy object forward. What is perhaps less obvious is that a conventional aircraft flies by the same principle; as we have seen (Chapter IV) the wings drive downwards, and thus impart momentum to, the particles of air beneath them and so experience a 'recoil' which keeps the aircraft aloft. This is the 'secret of the art of flying' for which Leonardo da Vinci was groping, and the only essential difference between the action of a rocket and an aircraft wing in this respect is that the one gets its recoil by generating gas and projecting it downwards independently of the atmosphere, while the other uses the air through which it moves for the same purpose.

*Quantitative Analysis of Rocket Motion.** The equation of motion of a rocket in its simplest form is now very easily obtained, especially if we agree to ignore gravity (that is, to disregard the weight of the rocket) and air resistance as a first approximation. Mathematically, this is the case of a rocket moving under 'no forces' or in 'empty space', but in plain language what is being done is to consider a rocket moving through air but with a thrust so large that weight and air resistance combined make very little difference to its motion. This is often the case with small high-speed military rockets.

Suppose the mass of the rocket at time t is M and that it is then moving with velocity V. Its momentum is MV, and since there are no external forces, this must remain unchanged throughout the motion. The rocket action redistributes this momentum between the rocket itself and the gases in the jet, and the problem consists of finding what changes occur in the motion of the rocket as a result of this redistribution. This is done by comparing conditions at the beginning and end of a short interval of time which, in the usual notation of the calculus, is called dt.

Suppose the constant velocity of the gas jet is V_E. In the short time dt a particle of mass dM is ejected. This particle already had a forward momentum VdM and now it acquires a backward momentum $V_E dM$ so that its net momentum is $(V-V_E)dM$. During the same short time the rocket increases its speed a little, say by dV, so that the momentum of the rocket becomes

$$\text{new mass} \times \text{new velocity} = (M-dM)(V+dV)$$

The total momentum being unaltered we can write for the system as a whole

* The reader who is not acquainted with the elementary ideas and notation of the calculus may omit this short section without endangering his understanding of the rest of the chapter.

$$(\text{original momentum} =)\ MV = (M - dM)\,(V + dV) + (V - V_E)dM\ (= \text{new momentum})$$
$$= MV + MdV - dMdV - V_E dM.$$

Dividing by dt and going to the limit (so that the second order small quantity $dMdV$ may be disregarded) we find

$$M\frac{dV}{dt} = V_E\frac{dM}{dt}$$

This is the *rocket equation* in its simplest form.

The quantity dV/dt is the acceleration of the rocket and dM/dt is the rate at which the fuel is being consumed. The product $V_E dM/dt$ is the thrust, so that in plain language this equation says that, in empty space,

$$\text{acceleration of rocket} = \text{thrust} \div \text{mass remaining}.$$

In the atmosphere, when both gravity and air resistance operate to slow down the rocket, terms representing the weight and the drag of the rocket must be subtracted from the thrust. In empty space, provided that the thrust can be maintained, a rocket can move with ever-increasing acceleration and there is no limit to the speed which can be attained in this way except that imposed by the amount of fuel carried.

AERODYNAMICAL PROBLEMS IN ROCKET DESIGN

The aerodynamical problems which arise in the design of rockets fall into two classes, internal and external.

Internal. The fuel carried by a rocket is always considerable and may even make up the greater part of the total weight, and since its sole purpose is to produce thrust, it is an advantage to develop a maximum thrust, and this at as early a stage as possible to avoid lifting unnecessary weight. The rocket equation shows that

$$\text{thrust} = (\text{rate of consumption of fuel}) \times (\text{velocity of the gas jet})$$

so that a large thrust can be obtained by burning the fuel very rapidly, or by speeding-up the gases in the jet, or both.

Solid fuel rockets traditionally burn gunpowder, but most military rockets of to-day employ one or other of the group of substances known as *propellants*. These belong to the wider class of 'explosives', but differ considerably from high explosives, such as dynamite, in their mode of action. Propellants (of which cordite is the most familiar example in this country) burn in the normal sense of that word, but they possess the characteristic property that the rate of burning increases with the pressure of the gases generated, so that while cordite burns fiercely but harmlessly in the open air, burning becomes extremely rapid and partakes of the nature of an 'explosion', when it takes place in a closed vessel, such as the firing chamber of a gun. In a cordite-operated rocket the gas pressure mounts rapidly once ignition is established, but quickly reaches an equilibrium value, determined by the design, when the rate of production of the gas exactly balances the outflow through the orifice. With such rockets the pressure in the tube is usually high, and since the gases themselves are at very high temperature, it is evident that any attempt to get a large increase in the rate of fuel consumption may easily result in the rocket bursting under the internal pressure, for the strength of a steel tube decreases rapidly at high temperatures.

In liquid fuel rockets pressure problems are generally not as serious as with solid fuel rockets, but the maintenance of a high rate of consumption of fuel implies engineering problems of a very high order. In V2, for example, the Germans had to develop and instal a special turbine delivering over 600 horsepower at about 5,000 revolutions per minute to work the pumps required to feed about $1\frac{1}{2}$ cwt. of liquid oxygen per second plus 1 cwt. of alcohol per second to the combustion chamber.* This was an extraordinary feat of engineering and

* Kooy and Uytenbogaart, *Ballistics of the Future* (1946).

the difficulties of increasing the rate of flow still further can easily be understood.

Having fixed on a practicable rate of fuel consumption the designer can turn his attention to the problem of getting the gases out of the rocket as quickly as possible, and for this he has to call in the science of gas dynamics. This means the study of the flow of gases at high speeds, taking into account thermodynamic relations, and therefore involves the phenomenon of compressibility, since large changes arise in the density of the gas. In compressible flow the unexpected is always happening, even in what appears to be the simple problem of high speed flow in a pipe. In incompressible flow the quantity of gas flowing through a pipe in a given time can be maintained constant by making the cross-sectional area of the pipe vary inversely as the speed of the flow. Conversely, if we want to increase the speed of flow while keeping the rate of discharge fixed, we must make the pipe contract. This all seems so obvious and 'commonsense' that it may come as a surprise to find that the reverse is true at very high speeds, that is, in these circumstances the pipe must be widened, not narrowed, to increase the speed of the gas flowing through it. This is because in gas dynamics not only does the pressure decrease as the velocity increases, but the density decreases also. To obtain a steady high-velocity stream of gas from a reservoir at high pressure it is necessary first to contract the stream in order to get the initial speeding-up, and then to allow it to expand as compressibility effects appear.

More precisely, suppose the combustion chamber of a rocket contains gas at high pressure. This represents a store of energy which has to be made available to drive the rocket, that is, the pressure energy must be converted into energy of motion, or kinetic energy. The gas will rush out of the chamber through any orifice which happens to be available, undergoing in the process a pressure-drop and acquiring a

velocity, so that the orifice is the means whereby the stored-up energy of the gas is released and transformed. An orifice which has been designed to perform this transformation in the most efficient manner is termed by engineers a *nozzle*, and a great deal of research has been directed towards perfecting nozzle design because of its importance in turbine construction.

Whatever be the shape of the nozzle, one condition must always be satisfied, namely the equation of continuity, or the proviso that the amount of gas which enters the nozzle in a given time must be exactly equal to the amount which leaves it in the same time – matter is neither created nor destroyed. We have encountered this equation before, when it performed the function of a watchdog on the solutions of the equations of motion, but here it is rather in the role of the dictator of the flow, for it is this condition which brings out the effects of the variable density. The continuity condition is simply

$$\text{(area of cross-section)} \times \text{(velocity)} \times \text{(density)}$$
$$= \text{quantity passing through} = \text{constant.}$$

It can be shown that this can be satisfied by making the mass flow through any cross-section of the nozzle depend simply on the ratio of the pressure in the combustion chamber to that at the cross-section, so that the nozzle must be shaped to bring about the correct reduction in pressure at every point, if smooth flow is to be achieved.

Suppose the designer decided that the best way of doing this would be to allow the pressure to drop uniformly from the chamber to the orifice. He can find the shape of nozzle which will bring about such a uniform pressure-drop quite easily, by using the fact that the required cross-sectional area is inversely proportional to the product of the density and the velocity, whose values are determined by Bernoulli's equation in the form appropriate to compressible flow. He would then

get the nozzle shape shown in Fig. 30 (*a*) in which the cross-section area first decreases to a minimum and then increases towards the outlet. Such a nozzle is called a *convergent-divergent* or *Laval type nozzle*. The cross-section of minimum area is termed the *throat*, and it is here that the speed of the stream reaches the *local velocity of sound*, which is the speed of sound appropriate to the temperature of the gas at that point, and is thus the velocity with which sound waves or other

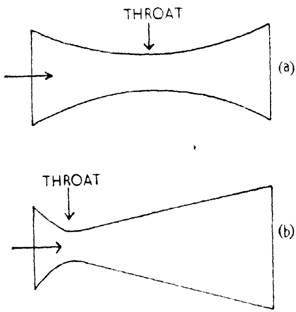

Fig. 30. Convergent-divergent nozzles.

small disturbances move relative to the gas, and not to the nozzle. After passing the throat the velocity continues to increase and becomes supersonic in the divergent portion of the nozzle towards the outlet.

The pressure at the throat is called the critical pressure* and it marks a curious phenomenon. Once the external pressure has beer lowered sufficiently to reach this pressure in the throat the rate of mass flow reaches its maximum and no further reduction of exit pressure will succeed in changing it,

* This should not be confused with the critical pressure of the physicist, which relates to the liquefaction of gases.

for from this point onwards the flow is supersonic and no external changes can ever be communicated to the throat. At this stage the combustion chamber loses contact with the outside world.

In practice the nozzle is not given the shape shown in Fig. 30 (*a*) but one more like that shown in (*b*), in which the diverging portion is made very much longer than the convergent part. If the divergent portion were omitted the flow would no longer be steady and smooth, but pulsating.

For the actual design of rocket nozzles it is necessary to reconsider some of the statements made earlier. The simple mathematics of the preceding section gave the result that the thrust is equal to the product of the rate of fuel consumption and the exit velocity of the gases, but this is only exactly true when the nozzle is so designed that the gases emerge at atmospheric pressure. The rocket problem has been oversimplified by treating it as a question of particle dynamics, whereas in reality it is a problem in hydrodynamics, and a closer examination shows that in the complete expression for the thrust an additional term appears, equal to the area of the orifice multiplied by the difference between the pressure in the jet and the external atmospheric pressure. This leads to some important conclusions. In the first place it can be shown that for a given value of the exit pressure, the thrust is a maximum when the external pressure is zero, so that *a rocket works most efficiently in a vacuum.* Secondly, if the exit pressure be varied, the maximum thrust occurs when the gases in the nozzle are allowed to expand until they reach the external pressure, so that theoretically a rocket nozzle should always be made long enough to achieve this expansion. In practice, designers, for a variety of reasons, prefer to use nozzles which are slightly shorter than the optimum, resulting in under-expansion and a slight loss of thrust, but with compensating advantages in other respects.

External. One of the most important aerodynamical problems in rocket flight is undoubtedly that of stability. Rockets are usually long slender bodies, and the first requirement is clearly that the weight shall be concentrated well forward or, in technical language, the centre of gravity, the point at which the weight acts, must be ahead of the centre of pressure, the point at which the aerodynamic forces act.

Stability in a shell is secured by rapid rotation, which produces powerful gyroscopic forces, and at first sight it may not be obvious why this method is not universally used in rockets also. Actually, some of the early rockets were spinners; this was so, for example, with the military rocket designed for the U.S. Army by William Hale about 1860, in which rotation was obtained by slanting vanes in the nozzle. The Germans also used small spinning rockets during the war, getting the rotation by replacing the normal single nozzle at the rear by a ring of small nozzles with a sideways tilt, giving a rotary force as well as a forward thrust. Most rockets, however, rely on the ancient device of fitting vanes at the rear to keep the projectile on its course, exactly as our forbears used feathers on their arrows.

It is not difficult to see why stabilization by rotation is not often used in rockets. Everyone knows that a top which is short and squat is much easier to spin, and much less affected by random disturbances when spinning, than one which is long and thin. It can be shown that the criterion for the stability of a rotating shell is the same as that for a top, or, in other words, a shell which has a large diameter and which is not too long can be made very stable in flight by the rotation imparted to it by the rifling in the barrel. A rocket tends rather easily to become long and narrow, and to stabilize such a projectile by rotation alone is like trying to make a pencil stay upright by spinning it on its point – it can be done, but an extremely high rate of spin is needed compared with that required for the

child's top. To get the necessary high rate of spin with a long rocket is always extremely difficult and sometimes impossible, and thus the designer falls back on vanes as the practical way of solving his problem.

The principle of the vane-stabilized rocket is extremely simple and intuitive; as soon as the rocket yaws, so that the relative air motion is not along the axis of the rocket, the vanes offer resistance to the air and provide a force which pulls the rocket back on its path. Vanes, however, have their disadvantages. They give rise to large wind effects and are responsible for one of the distinctive features of rocket flight, namely, the eccentric behaviour of a rocket in a wind which is blowing across its line of flight. When a shell meets such a cross-wind it does what would be expected, that is, it drifts a little downwind and the gunner has to make a simple correction for this. A vaned rocket, on the other hand, moves *against the wind* during the period of burning and *with the wind* when burning has ceased. This odd behaviour is quite simply explained. A cross-wind has a much more powerful effect on the vanes than on the rest of the body, so that the tail moves downwind and the rocket is steered into the wind. This would not matter if the thrust were not operative, but during burning the thrust pushes the body in the direction in which the nose is pointing, which is slightly against the wind. When burning has ceased the effect of the wind is simply to translate the whole rocket downwind, just as in the case of a shell.

Rockets as Projectiles. The war saw the latest and most impressive revival of the rocket as a projectile and it is of interest to examine the arguments for and against the free rocket as a weapon. Excluding giant long-range rockets such as V2, which are in a class by themselves, and all 'guided' weapons, the rocket principle at first sight appears to offer certain definite advantages over conventional artillery.

Perhaps the most striking feature of the rocket is the entire absence of recoil on the launching gear, which means the elimination of much heavy mechanism which has to be included in a modern field piece. To launch a rocket all that is needed is a pair of rails or a light tube to support it for the first few feet of its flight, and such equipment need only be strong enough to carry the weight of the rocket and to remain stable during the fraction of a second when the rocket is moving off. There is no energy of recoil left behind to be absorbed and the elaborate buffer system of a gun is not required. A second great advantage is that, unlike a gun, the projector does not become ineffective through wear. A gun barrel has to be made with great accuracy and designed to stand pressure of many tons per square inch, and the heavy and continuous wear of the barrel, especially with high-velocity guns, means a steady and irrecoverable loss of accuracy. The rocket projector does not call for such fine workmanship and is virtually everlasting, since the forces on it are negligible compared with those in a gun barrel.

So much for the advantages; the disadvantages are, however, numerous and serious. The blast from the jet is a very unpleasant feature and an intensely hot flame roaring out of the nozzle at a speed in the neighbourhood of 4,000 miles per hour is bound to throw up a large cloud of stones and dirt when near the ground. The main trouble, however, is the inherent lack of accuracy of a rocket compared with a gun. This inaccuracy arises from many sources, such as the inevitable manufacturing errors in the alignment of the nozzle, lack of straightness in the tube and so on, and also because of the susceptibility of vaned unrotated projectiles to meteorological influences. It has already been explained that the wind near the ground is highly turbulent, so in this layer both speed and direction vary considerably from second to second and also with height. The effect of such variations on the motion of a

projectile obviously depends not only on the magnitude of the oscillations but also on the time spent in traversing the layer of the atmosphere in which they occur. A shell is fortunate in that, starting with its maximum velocity at the muzzle, it spends the least possible time in this highly disturbed layer and experiences the bulk of the meteorological effects at the top of its trajectory, where conditions are usually extremely steady. A rocket starts slowly and does not reach its maximum speed until the end of burning, so that it spends a relatively long time in the highly turbulent lower layers and consequently is very susceptible to the effects of the unpredictable oscillations in wind which occur near the ground. Data which have been released for American military rockets* show that even with the best designs the dispersion is between four and five times that obtained with a gun, and also that most of this dispersion occurs during the burning period. It follows that free rockets are admirably suited for the mass attack of area targets, but cannot compete with guns against small targets, which is exactly where Congreve left the matter.

JET-PROPELLED AIRCRAFT

Let us now turn awhile from the rocket proper and consider the jet-propelled aircraft. It has been explained in the previous chapter that the propeller becomes less and less efficient as the blade-tip speed nears that of sound, so that the quest for yet higher speeds of aircraft means that other forms of propulsion must be investigated. During the war the jet-propelled fighter aircraft made its appearance and the information released so far about these machines shows that speeds in the vicinity of that of sound have already been attained.

A jet-propelled aircraft differs essentially from the rocket in

* Rosser, Newton and Gross, *The Mathematical Theory of Rocket Flight*, New York, 1947.

that it makes use of the atmosphere for its propulsion, whereas the rocket, as we have seen, is entirely independent of the surrounding air in this respect. Essentially, a jet-propelled machine consists of a conventional aircraft fitted with one or more ducts, open to the air at both ends. As the machine moves forward, the air enters the ducts into which there is built a device, usually a *gas turbine*, which greatly accelerates the stream and throws it out at the rear in the form of a high-speed jet. The gas turbine thus supplies additional momentum to the air and therefore generates a thrust which drives the machine forward. The action is like that of a rocket in that the thrust is derived from the formation of a high-speed jet, but is even more like that of a propeller in that what has been done is to accelerate an air stream, just as the propeller accelerates the air passing through its disc.

The advantages to be obtained in this way require careful consideration, but one is immediately obvious. It is of little use for an aircraft designer to strive for ultimate perfection in the shape of the wing if the propeller sweeps over it a mass of highly disorganized air, and the jet scheme has the very considerable advantage that the wing is always meeting smooth air. It is for this reason that laminar-flow wings (Chapter IV) are specially suitable for jet-propelled aircraft.

The real advantages of the turbo-jet do not appear until the machine is travelling very fast. Curves comparing the propulsive efficiencies of propellers, jets and rockets tell an interesting story.* At a height of 20,000 feet the propeller is working at its highest efficiency when the air-speed of the machine is between 200 and 400 miles per hour, and it attains this state very quickly, being not very much below maximum efficiency at 100 miles per hour. The turbo-jet compares badly with the propeller in the lower speed range – at 300 miles per hour, for example, its efficiency is assessed at about half that of the

* *v.* Fedden, *Journal of the Royal Aeronautical Society*, 1944.

propeller, but it enters into its own at the top speeds. At about 550 miles per hour the efficiencies of the two modes of propulsion are about the same, but from this point onwards propeller efficiency falls off swiftly, while the turbo-jet is still rapidly gaining in efficiency. Much inferior in efficiency to both of these is the rocket; even at 600 miles per hour it is still woefully below both the propeller and the turbo-jet, but it has one great advantage; it can fly where both the turbo-jet and the propeller fail for lack of air, and the highest speeds of all are those attained by the rocket.

The study of methods of jet-propulsion is outside the plan of this book, but some mention should be made of what is perhaps the simplest aircraft yet constructed, the German flying-bomb (V1). The propulsive unit of this device, which is carried above the fuselage and aft of the short and extremely simple wings, consists mainly of a slightly tapered sheet steel tube, the forward portion of which forms the combustion chamber. As the aircraft moves forward, the dynamic pressure of the air on the forward end of the steel tube forces open an assembly of spring-leaf flap valves; a quantity of fuel is then injected into the chamber and the mixture fired by an electric spark. The resultant gas pressure is large enough to close the spring-leaf valves on the forward end so that a high-speed jet of gas rushes out of the rear end, giving the machine a forward impulse. As the gas emerges, the pressure in the combustion chamber falls, the valves open and the whole process starts again, giving the characteristic 'motor-cycle' sound (frequency about 40 per second) which Londoners soon learned to recognize. In this way nearly 600 horse-power was developed by this primitive engine. It is hard to conceive of anything simpler and more suitable for rapid production in time of war, and as a weapon it failed completely once the initial surprise was over. An aircraft which flies straight and level and at a constant speed and height is the one target above all else for

the anti-aircraft gunners, who with the aid of radar and the electronic fuze reaped a rich and joyous harvest.

GIANT ROCKETS AND THE PROBLEM OF ESCAPE FROM THE EARTH

The most exciting thing about a rocket is that, alone of all flying-machines, it offers a chance of escape from the gravitational field of the earth and of reaching other worlds. It is appropriate, therefore, to bring this short account of man's conquest of the air to a close by examining the possibility of even greater feats.

It is, of course, chiefly the *possibility* and not the *practicability* of space-travel which can be discussed here, and it is as well to begin by making clear what is meant by this statement. Mechanical flight has been possible since the dawn of time, but was not practicable until the internal-combustion engine had been invented; it is possible to-day to release from a few bars of uranium enough energy to drive the *Queen Mary* across the Atlantic, but it is not yet practicable to do so. When the scientific world unites in declaring that true perpetual motion is impossible in a mechanical system it means that this recurrent dream of inventors cannot ever be realized, not because man has insufficient wit to solve the problem, but because we are confident that the universe is not built that way. As far as life on this planet is concerned, a 'perpetual motion' machine can be constructed at any time by harnessing, for example, the energy of the tides, but this in no way refutes the scientific dogma, for in some remote epoch even the tides will cease to ebb and flow. Interplanetary travel by rocket is possible in the sense of not being ruled out by some fundamental principle like the Second Law of Thermodynamics, but it has not yet been shown to be practicable.

We can, however, get rid of some superstitions at the start. Jules Verne wrote an entertaining account of a vast gun which propelled a shell to the moon, and H. G. Wells imagined a mysterious material called *cavorite*, 'opaque to gravity', for the same purpose. The first of these is certainly impossible at the present time, because no matter how long the barrel of the gun be made, no known propellants could expel the shell at the enormous speed required to carry it beyond the reach of gravity. The second is outside the domain of science, because as far as we know gravity is utterly unlike radiation and is probably best described as a 'property of space' which we cannot affect in any way. This need not impair the reader's enjoyment of *The First Men in the Moon;* Wells had to find some way of getting to an imaginary world (his moon is a satellite unknown to astronomy) in which he could discuss his sociological problems in peace, without having to invent a machine capable of reaching the staggering speed required, and he did this very neatly by eliminating gravity, the cause of all the trouble.

A preliminary examination of the intriguing problem of space-travel need not involve difficult mathematics and is little more than a pleasant exercise in arithmetic. The investigation becomes difficult only when the engineering aspects enter.

The Velocity of Escape. All problems of space-travel start with the same simple calculation, namely the determination of the speed which must be reached in order to get away from the earth's influence. The gravitational pull of the earth, like that of any other body, decreases inversely as the square of the distance from its centre and, from astronomical and other studies, the constants in the equation are known to a high degree of accuracy. To reach a point in space at which the earth's gravitational field is negligible means that the body must provide or be given sufficient energy to attain a certain speed, known as the *velocity of escape*. This speed is very easily

calculated because it is simply the velocity with which a body, starting from rest at the outer limit of the earth's field (mathematically, 'at infinity') would strike the earth, and is equal to 36,700 feet per second, or 25,000 miles per hour or approximately seven miles per second. This calculation, it is true, ignores such factors as the resistance of the air, the speed of the earth's rotation and the slight gravitational pull of the moon, but the combined effects of all those have relatively little influence on the figure given above.

The problem of space-travel thus amounts to designing a rocket capable of reaching a speed of at least 25,000 miles per hour and this brings us immediately to the hard core of the problem, an inherent property of all rockets and which accounts for the fact that, unlike other projectiles, a very high-speed rocket must be very large, if only those fuels known to us at present are to be used. This conclusion rests on the so-called *Law of Mass-Ratio.*

The Law of Mass-Ratio. To deduce this law we must, at this stage, make a brief return to the rocket equation (p. 173) but now subtracting from the thrust the pull due to gravity. That is, the equation becomes,

$$M\frac{dV}{dt} = V_E\frac{dM}{dt} - Mg,$$

where M is the mass of the rocket at time t, V_E is the jet speed and g is the acceleration due to gravity, so that Mg is the weight at any instant. If we suppose g to be constant, or equal to the mean value of the gravitational acceleration over the path of the rocket when the thrust is operative, the solution of this simple equation leads to an equally simple expression

$$\frac{\text{Mass of rocket} + \text{fuel}}{\text{Mass of rocket alone}} = e^{(V_F + gt)/V_E},$$

where e is the base of natural logarithms ($=2{\cdot}7218$ ...), V_F is the final or 'all-burnt' speed of the rocket and t is the time

taken to consume all the fuel. The resistance of the air has been omitted, since it does not affect the order of magnitude of the result, and in this discussion we are concerned only with orders of magnitude.

This is the equation which spells the doom of any hope of a quick and easy solution of the problem of escape from the earth. The ratio (mass of rocket+fuel)/(mass of rocket alone) sets the problem for the designer; if this ratio is at all high, it means that the designer is confronted with a very difficult task, for he has then to allot a large proportion of the total weight of the rocket to the fuel, or, in other words, he is severely limited as regards the case, mechanism and pay-load because of the demands for fuel capacity. This is because of the rapid increase of the ratio as the index increases, as the following table shows:

Value of $(V_F + gt)/V_E$	*Mass Ratio*	*Percentage of total mass to be carried as fuel*
½	1·65	40 per cent.
1	2·72	63 ” ”
2	7·39	87 ” ”
3	20·1	95 ” ”
4	54·6	98·7 ” ”
5	148·4	99·4 ” ”

This table makes it clear that once $(V_F + gt)/V_E$ reaches a value in excess of 2, the engineering problem becomes extremely formidable, and one does not need to be an expert in rocket design to realize that the higher values of the index rule out any hope of a practical solution. Thus every effort must be made to keep $(V_F + gt)/V_E$ as small as possible, despite the fact that V_F is fixed at 25,000 miles per hour.

Only two quantities remain under the designer's control, the time of burning (t) and the jet speed (V_E), and it is clear that the beneficial effect of reducing t is very limited. The mass-ratio law shows, however, that long times of burning are generally adverse. The dangers and difficulties attendant upon achieving a very short time of complete combustion are evident, and the designer will be doing very well if he can keep t as low as a matter of a few minutes, say 200 or 300 seconds. His only real hope of achieving success lies in increasing the jet velocity.

Jet Speeds. From a study of the chemical reactions which yield large amounts of energy it is possible to calculate theoretical maximum jet speeds with great accuracy, and if a reasonable estimate be made of the inevitable losses which occur in real fuel systems, the possibilities can be stated with considerable certainty. This has been done recently by F. J. Malina and M. Summerfield of the California Institute of Technology in a paper discussing the potentialities of rocket flight in the problem of escaping from the earth.* Their conclusions may be summarized as follows: nitromethane (a mono-fuel), red fuming nitric acid + aniline, and (concentrated) hydrogen peroxide + methyl alcohol (bi-fuels) are all practical sources of energy, which can be expected to give jet speeds of the order of 5,000 miles per hour. The liquid oxygen +ethyl alcohol combination used in V2 gives a slightly higher figure, about 5,500 miles per hour. A liquid oxygen+liquid hydrogen reaction, if it could be used, would provide a jet with a speed just over 8,000 miles per hour. (These figures take into account the effect of reduced air pressure at the greater heights.)

Thus even the best of fuels implies jet speeds far below that of the velocity of escape, and even in the case of the liquid oxygen + liquid hydrogen reaction only about 4 per cent of

* *Journal of the Aeronautical Sciences*, Vol. 14, August 1947.

the total weight of the rocket at the start would be available for the rocket body, fuel-tanks, pumps, pipes, combustion chamber and pay-load. The conclusion is inescapable; the mass-ratio law makes the problem of escape utterly impracticable for a straightforward rocket using known fuels.

Before leaving this aspect of the problem it is of interest to see what the Germans achieved when they built the world's largest rocket, V2. The jet velocity of this rocket was probably about 4,700 miles per hour and the time of burning was nearly 60 seconds. To get a final speed of about 3,500 miles per hour, the German designers had to achieve a mass-ratio of about 2·8, which they did by making the total weight of the rocket about 12½ tons, of which some 8 tons was fuel. This in itself was a most remarkable feat of engineering.

The Step-Rocket. Some scheme other than that of a straightforward rocket is therefore essential if a final speed of 25,000 miles per hour is to be realized. A serious drawback of the conventional rocket is that it is obliged to carry with it to the end of the journey the empty fuel-tanks, pumps and combustion chamber when these have long ceased to play any useful part. It was suggested by Oberth and other early investigators that a practicable space-ship could be built by making a rocket capable of discarding these useless masses when they have done their job. That is, the rocket should be made up of a number of 'steps', each equipped with a propulsion unit and carrying a fraction of the total load of fuel, which can be cast-off when exhausted.

This scheme has also been examined in detail by Malina and Summerfield (*loc. cit.*). A first question concerns the number of steps to be used, and a second is whether the steps should be operated in rapid succession or whether the rocket should be allowed to 'free-wheel' between successive firings. Malina and Summerfield find that to get any considerable advantage the number of steps can hardly be less than five but

need not be more than ten, and that 'free-wheeling' is a definite disadvantage, that is, a step should be operated once the preceding step has been exhausted and discarded.

At this point the arithmetic becomes exciting, for we are beginning to approach the practicable. Suppose it were planned to project a very modest mass, say ten pounds, away from the earth, using a step-rocket with the nitric acid + aniline fuel combination. A five-step rocket to do this would weigh, fully loaded, about 367 tons at the start and the projectile would be a vast rocket about 130 feet long with a diameter of 13 feet. (V2 was about 43 feet long and had a diameter of about 5½ feet.) Most of the mass, about 329 tons, would be contained in the first step, which would be exhausted of fuel and discarded after about 40 seconds. The second step would have a mass of about 34 tons, the third about 3½ tons and so on, until the final projectile, which would shake itself free of the clutches of the earth, would be a relatively small affair, having a mass of less than 100 lb. including its precious 10-lb. cargo. This scheme is perhaps just practicable, but hardly attractive. A ten-step rocket would naturally entail much greater difficulties in design, but would show an enormous saving in the initial mass and size. Malina and Summerfield estimate that, for the same ten-pound payload, the all-up weight of such a projectile would be about 60 tons at the start; its length would be about 70 feet and its diameter about 7 feet, that is, nearly twice as long as V2 and about half again as thick. This is much nearer practicability.

The limit of performance with known fuels and ordinary chemical reactions is probably reached with a ten-step rocket using the liquid oxygen + liquid hydrogen combination. With such a fuel system it is estimated that a ten-step rocket weighing only 3½ tons at the start (and thus much lighter than V2, although it would have to be just about as large, because of the more elaborate structure needed to deal with liquid

hydrogen in bulk) could carry a payload of ten pounds beyond the earth's gravitational field. If the payload were increased to 100 pounds the initial mass of the rocket would have to be increased by a factor of seven.

The conclusion therefore is that it is certainly possible, and even may be practicable to-day, to project a small mass into free space, but the practicability really depends upon the successful solution of problems of a type which engineers are only beginning to study. Several writers have gone as far as examining the possibility of manned rockets undertaking journeys around the moon and even to Mars and back, but at this stage such investigations are purely speculative and can hardly be taken seriously, although they make entertaining reading.

The thought which no doubt immediately arises in the reader's mind is that the ultimate solution of the problem of the penetration of space must lie in the use of nuclear energy. This may well be so, but it should be made clear that even this is far from being straightforward. Nuclear energy, fully realized, would relegate the mass-ratio law to an insignificant second-order effect, but this is looking beyond anything which is justified at present. A more practicable suggestion for obtaining high jet velocities without prohibitively large rockets is by heating hydrogen to a high temperature with the energy obtained from nuclear sources, but to get really high speeds means some extraordinary problems for the designer. Theoretically, for a chamber temperature of 11,000° F. (about that of the surface of the sun) it is possible to obtain a jet with a speed of 28,000 miles per hour, considerably above that of the velocity of escape, but the engineering difficulties involved in the design of rockets with such jets are perhaps best left to the imagination.

The science of flight has traversed the road from the spear of the primitive hunter to the supersonic aircraft and the giant rocket, and man has conquered the air and may yet win a like dominion over space. It may justifiably be asked, to what end – but that is not a question which this book attempts to answer. Whether the aeroplane is a blessing or a curse is immaterial to the science from which it springs; it is enough that there are new facts to find, fresh problems to solve, and minds eager to accept the challenge which they bring.

THE END

APPENDIX

The Tools of Aerodynamics

A FIRST requirement for accurate experimental work in aerodynamics is that the investigator shall be able to produce a flow of air past a body and to control that flow within wide limits. It is not difficult to obtain a satisfactory flow on a small scale, but model work frequently necessitates making rather large volumes move quickly, and it is here that the main difficulties enter.

Nearly all the early work on experimental aerodynamics was done by moving the test body through the air, whereas the modern method is to make air flow past a stationary body. The simplest way of making a body move through the air is to drop it from a height; thus Newton used the method of free falling in his investigations on the resistance experienced by spheres (Chapter III). This type of investigation reached its highest state of development in the hands of the French engineer Eiffel, who in the early years of this century carried out a long series of experiments on the tower which bears his name. The velocities which can be reached in this way are limited and there are other obvious disadvantages.

Cayley, and later Lilienthal, used the *whirling arm*, the scientific counterpart of the roundabout of the fair, for the same purpose. A large whirling arm is still used for certain research work at the National Physical Laboratory, Teddington. The method is attractive in many ways, particularly for absolute measurements at low speeds, but has serious limitations and many disadvantages. After a few revolutions of the arm the air in the building is set in motion and these irregular induced currents can be very troublesome.

Wind Tunnels. The greater part of experimental work on aerodynamics is done in *wind tunnels.* The earliest true wind tunnel appears to have been that constructed in 1903 by Sir Thomas Stanton at the National Physical Laboratory. This was a rather crude affair, consisting of little more than a chimney through which air was drawn by a fan. A few years later Riabouchinsky in Moscow and Prandtl in Göttingen produced tunnels of types which have since become standard in aerodynamical laboratories all over the world.

Modern wind tunnels operate either with air at atmospheric pressure or with compressed air, and most tunnels are of the first type. They can be either of the *straight-through* or *closed-return* patterns. The straight-through tunnel is simply a large pipe through which air is drawn by a propeller, but precautions are taken to ensure that over a certain length of the pipe the air flow is as smooth and as uniform as possible. The inlet consists of a large trumpet-shaped mouth, leading to a honeycomb filter and a contraction, after which comes a parallel-sided portion, usually rectangular in shape, called the *working section,* in which the models and test bodies are placed. After the working section the tunnel diverges gently to the propeller housing at the outlet.

The purpose of the trumpet-shaped entrance is to ensure that the air enters at low speed and with the minimum of disturbances. Any large-scale swirling motions or cross-currents are broken up and reduced by the honeycomb, while the contraction serves to increase the speed and also to reduce any irregularities along the direction of motion. In the working section itself the air flow should be straight and smooth and with only a small and regular pressure-drop. The divergent cone (called the diffuser) slows down the stream and restores the air pressure to nearly its original value, so that in a well-designed tunnel the propeller has only to increase the pressure in the stream by a small amount in order to maintain a steady flow.

In the straight-through type of tunnel the air emerging from the propeller disc has to find its way back to the inlet via the laboratory, but a considerable saving in both power and space can be made by using the closed-return system in which the air is circulated from the outlet back to the inlet by a duct. Such tunnels can be of the closed or open-jet types. In the open-jet type, introduced by Eiffel, the parallel walls of the working section are cut away for a short distance and the whole enclosed in a large reasonably air-tight chamber. Open jets are obviously more convenient for many purposes; since it is possible to make adjustments to the model without difficulty.

Atmospheric pressure wind tunnels vary considerably in elaboration, size and range of speed, but need not be very large or expensive to construct if only small models are used. A straight-through tunnel with a working portion of 1 foot square cross-section would need to be about 20 feet long and would require a motor of about 5 h.p. to reach a speed of about 60 miles per hour in the working section. Some of the best work in aerodynamics has been done in tunnels of quite modest size and power.

In a normal wind tunnel high Reynolds numbers can only be reached by using high speeds or large models, and although some very large tunnels have been made (e.g. that at Langley Field, U.S.A.) it is rarely possible to attain the full Reynolds number appropriate to a modern aircraft in normal flight. The power demanded from the motor which drives the propeller varies as the cross-sectional area of the working section and as the cube of the speed, so that as the size of the test body (and hence of the tunnel) or the air speed rises, the power of the motor must be correspondingly increased and may soon reach prohibitive values. An alternative way of reaching a high Reynolds number without increasing either the size of the tunnel or the air speed beyond economic limits is to

reduce the kinematic viscosity of the air. This can be done by increasing the density of the air by compression – a twenty-fold increase in pressure means about the same increase in *Re*. Naturally, such variable-density tunnels are more expensive to construct and less easily used than those working at atmospheric pressure, since all measurements and adjustments must be made by remote control. The Langley Field Variable-Density Tunnel is enclosed in a steel tank 15 feet in diameter and 35 feet long, capable of withstanding a pressure of at least 20 atmospheres, and with this arrangement a Reynolds number of just over 3,000,000 is obtained with a model aerofoil of chord of 5 inches. A 200 h.p. motor is used and a speed of about 55 miles per hour is reached in the working section.

In all the above tunnels the speed of the air is such that compressibility effects are negligibly small. High-speed tunnels, in which the velocity of the stream approaches that of sound, call for very special technique. It is extremely difficult to produce a steady supersonic stream of useful dimensions at atmospheric pressure because of the enormous power required, and the method usually used is to generate a high-speed low-pressure jet for a short time only. Stanton produced a high-speed tunnel of this type in 1930, mainly for research on ballistics. He discharged a large volume of highly compressed air (actually that obtained from the N.P.L. variable-density tunnel) into the atmosphere through a small (3-inch diameter) convergent-divergent nozzle and in this way reached a Mach number of 3. A rather different type of high-speed tunnel is that which uses a high-pressure jet of air to induce a much larger high-speed flow of air from the atmosphere. Such induction-type tunnels exist at the National Physical Laboratory and at Langley Field. The Royal Aircraft Establishment, Farnborough, has an elaborate closed-in type of high-speed tunnel, in which the pressure can be varied over

wide limits. The working section is 10 feet $\times$ 7 feet, through which the 4,000 h.p. motors can drive the air at a Mach number of 0·8, giving a maximum Reynolds number of 12,000,000 with a model aerofoil of chord of 18 inches. In addition the air circulating in the tunnel can be maintained at any temperature between $+10°$ C. and $-5°$ C. by means of refrigerating plant. This elaborate and costly installation bears out the statement made earlier in this book that experimental high-speed aerodynamics can only be done on a national scale.

The Aerodynamic Balance. Wind tunnels are normally used to study aerodynamic force, and this again calls for special techniques. In model work there are two distinct major problems to be solved, those of mounting the model and of measurement of force. It is obviously essential that the air flow around the test body be as free as possible from artificial disturbances, and this implies also that the model must not be in close proximity to the walls of the tunnel, so that a suspension of some kind is called for. The design of a suitable suspension or support is not always easy. It would be of little use, for example, taking pains to construct an exact copy of a finely streamlined body, such as an airship hull or aircraft wing, and then mounting it in a tunnel with a massive support. The difficulty of using models is increased by the feature of fluid motion known as interference, which means that not only is the drag or lift of the test body affected by the proximity of the mounting, but in addition the resistance offered by the mounting is also affected by the nearness of the test body. Difficulties of this sort can be minimized by placing the support in a position where its effect is least, e.g., in the eddying wake, but the whole problem of mounting a test body is one which requires considerable skill and, above all, experience on the part of the investigator if reliable results are to be obtained.

The components of aerodynamic force on a body are

measured by the *aerodynamic balance.* Essentially this is an ordinary steelyard modified to give great accuracy and sensitivity. Aerodynamic balances have increased in complexity with the development of the science, but the basic principle remains the same, and the reader is referred to one or more of the specialized text-books given in the Bibliography for the exact details of arrangement and construction, which are frequently complicated.

Measurement of Pressure. In aerodynamic work it is essential to know both the static pressure (p_s) and the dynamic pressure ($\frac{1}{2}\rho u^2$) in the air stream. In the case of flow past a solid body, such as an aircraft wing, the static pressure is easily measured by drilling a small hole in the surface of the body and connecting this to a pressure gauge. The air flows past the hole but causes virtually no motion inside the hole itself. (There is a slight suction effect due to viscosity but this can be made negligibly small by decreasing the diameter of the hole.) The same principle is used to measure the static pressure in a free air stream, by inserting into the stream a thin closed tube placed parallel to the flow but with a number of very small holes drilled in the side. Such a device is called a *static tube*, and if properly designed with a rounded nose and smooth body will show no significant change in pressure with the speed of the stream, provided it is accurately positioned parallel to the direction of flow.

The dynamic pressure has already been defined (Chapter III) as the pressure exerted by a stream when it is brought to rest by a solid body. If a smooth tube of small diameter is placed with its open end facing up-stream and with the other end connected to a pressure gauge, Bernoulli's theorem shows that the pressure recorded in the stagnant air of the tube is the sum of the static and dynamic pressures. Such a tube is called a *pitot tube*, after the French scientist who first introduced it. If a pitot tube be connected to one side of a pressure gauge

and a static tube to the other, the gauge will show the difference between the pressures in the tubes

$$\text{or} \quad \underset{\text{(pitot)}}{p_s + \tfrac{1}{2}\rho u^2} - \underset{\text{(static)}}{p_s} = \underset{\text{(dynamic)}}{\tfrac{1}{2}\rho u^2}$$

that is, a device which combines both the pitot and static tubes can be made to measure dynamic pressure only. Such an instrument, consisting of a pitot tube surrounded by a static tube, is called a *pitot-static tube* and is one of the most familiar tools of aerodynamic research.

Pitot, static and pitot-static tubes are extremely useful for a wide range of problems because they can be made small and therefore suitable for exploring thin strata of air, such as boundary layers. To obtain full advantage from these devices necessitates the use of very sensitive pressure gauges, usually those termed *micromanometers*. The most elementary form of pressure gauge is a U-tube and the most sensitive micromanometer in general use, the Chattock-Fry gauge, is simply a glorified U-tube. The sensitivity of this instrument is such that it can measure changes of pressure as small as a few hundred-thousandths of an inch of water. How minute such changes of pressure are can be appreciated when it is realized that the normal surface pressure of the air, about 15 lb. per square inch, is equivalent to about 34 feet of water.

Measurement of Velocity. The dynamic pressure, $\frac{1}{2}\rho u^2$, depends only upon the density of the air and the velocity of the stream. This means that a pitot-static tube can be conveniently used to measure the speed of a stream, and the air-speed indicator of an aircraft is usually operated by a pitot-static tube mounted well out on the wing. For high flying a correction is needed to allow for changes in the density of the air from the standard density, and a correction of the same type enters when compressibility effects appear as a result of high speed. At 500 miles per hour, for example, to ignore

compressibility would lead to an error of about 10 per cent. in the indicated air speed.

The response of a micromanometer is fairly sluggish and such devices cannot be used to follow rapid fluctuations, such as those which occur in turbulent flow. To obtain information regarding such oscillations requires the use of an entirely different type of instrument, the *hot-wire anemometer*, which depends on the fact that the electrical resistance of a wire varies with its temperature.

In practice a short length of very fine platinum wire is maintained just below red heat by passing through it a small current of electricity. When placed in stagnant air the system immediately attains a balance in which the amount of heat leaving the wire by conduction and radiation exactly equals the amount of electrical energy supplied. If a stream of air moves over the wire, additional heat is taken away by forced convection, and to restore this balance more energy must be supplied from the battery. The usual arrangement is to make the wire part of a Wheatstone bridge, which is first balanced in still air. In moving air, the out-of-balance current passing through a sensitive galvanometer connected across the bridge is a measure of the speed of the air flow.

The hot-wire anemometer is an invaluable research tool in virtue of its ability to follow faithfully very rapid changes in air speed and also because the very fine wire employed makes for a minimum of disturbance to the flow. With an inertialess recorder such as a cathode-ray tube and a camera it is possible in this way to obtain a faithful record of the intricacies of turbulent flow. There are, however, certain disadvantages. There is a pronounced fall in sensitivity at the higher speeds and the fragility of the wire is often a nuisance, but the greatest drawback is undoubtedly the failure of the wire to retain its characteristics over any lengthy period, so much so that for accurate work it is necessary to recalibrate the instrument at

very frequent intervals. The cause of the change in the 'still air' resistance is uncertain, the most probable explanation being dust settling on the wire.

Making Air Flow Visible. A great deal of ingenuity has been expended on ways of making the pattern of air flow visible. The most obvious method is to employ smoke, either by generating a cloud near the inlet of a wind tunnel or by coating the surface of the model with a substance (such as titanium tetrachloride) which fumes when brought in contact with moist air. Many of the finest photographs of 'aerodynamics' are, however, really photographs of motion in water, made visible by sprinkling the surface with a fine powder, such as aluminium or lycopodium particles.

The study of high-speed flow has produced some very beautiful methods of rendering flow visible, depending on the fact that light is bent, or refracted, by changes in density. The spark-shadow system used in ballistics has already been described (Chapter VI); this requires only the simplest optical arrangement, but can be used successfully to reveal only very sharp changes in density such as those found in shock waves. Two other methods, the *schlieren system* and the *interference method*, are also in common use, especially the first named. Both require fairly complicated optical systems and so will not be described here in detail. Each system, and especially the second, is capable of revealing much smaller changes in density than the spark-shadow method.

The schlieren system uses either lenses alone or in combination with a concave mirror, similar to those used in small astronomical reflecting-telescopes. The essential principle is that a beam of light is made to pass through the air stream and also through an optical system containing a sharp-edged stop or stops before striking the screen or photographic plate. If the air is of uniform density throughout, the screen is evenly illuminated, but if regions of variable density occur, the rays

which pass through such areas are refracted. The system is arranged so that where the air is compressed, the refracted rays strike the stop and are blocked or reduced in intensity, and where the air is rarefied, those rays which were previously held up by the stop are now allowed to reach the screen, giving a brighter illumination. The resulting photograph is rather like a scene taken in bright sunlight, with patches of brilliant illumination alternating with shadows of varying intensity, thus indicating not only the areas of rarefaction (light) and compression (shade) but also giving in the graduation from light to shade a measure of the density gradient. Quite apart from high-speed flow, the schlieren system has produced some very fine photographs of convection currents from hot bodies, another case in which air flow is accompanied (or rather, is due to) changes in density.

The interference method (due to Mach) uses an interferometer to set up the familiar 'interference pattern' of dark and light strips (rather like a footballer's jersey) in the air at rest and then studies the deformation of the pattern caused by density changes in the moving gas. This is by far the most sensitive method yet devised. Strictly speaking, all fluid motion is 'compressible' and feeble density changes take place even in flow which is normally regarded as 'incompressible'. The interference method reveals density changes around solid bodies at quite low speeds, and in fact the method is really too sensitive for ordinary use; it is extremely difficult to eliminate all random fluctuations of density in a volume of air and to leave only those which are significant for the purpose in mind. Apart from this, the pictures are not always easy to interpret in detail and some extremely bizarre patterns, almost 'futuristic' in appearance, can be obtained from relatively simple flow.

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